Hypothesis Testing

Steps for Hypothesis Testing



Step 1: Formulate the Hypothesis

- A null hypothesis is a statement of the status quo, one of no difference or no effect. If the null hypothesis is not rejected, no changes will be made.
- An **alternative hypothesis** is one in which some difference or effect is expected.
- The null hypothesis refers to a specified value of the population parameter (e.g., μ , σ , π), not a sample statistic (e.g., \bar{X}).

Example of a Hypothesis Test

For the data in Table 15.1, suppose we wanted to test the hypothesis that the mean familiarity rating exceeds 4.0, the neutral value on a 7 point scale. A significance level of $\alpha = 0.05$ is selected. The hypotheses may be formulated as:

 $H_0: \ \mu \le 4.0$ $H_1: \ \mu > 4.0$ $t = (\overline{X} - \mu)/s_{\overline{X}}$ $S_{\overline{X}} = 0.293$

 $t_{CAL} = (4.724 - 4.0)/0.293 = 2.471$

One Sample : t Test

- •The df for the *t* stat is n 1. In this case, n 1 = 28.
- •The probability assoc with 2.471 is less than 0.05. So the null hypothesis is rejected
- Alternatively, the critical t_{α} value for a significance level of 0.05 is 1.7011 •Since, 1.7011 <2.471, the null hypothesis is rejected.
- •The familiarity level does exceed 4.0.

•Note that if the population standard deviation was **known** to be 1.5, rather than estimated from the sample, a *z* **test** would be appropriate.

Step 1: Formulate the Hypothesis

- A null hypothesis may be rejected, but it can never be accepted based on a single test.
- In marketing research, the null hypothesis is formulated in such a way that its rejection leads to the acceptance of the desired conclusion.
- A new Internet Shopping Service will be introduced if more than 40% people use it:

*H*₀: $\pi \le 0.40$

*H*₁: $\pi > 0.40$

Step 1: Formulate the Hypothesis

- In eg on previous slide, the null hyp is a one-tailed test, because the alternative hypothesis is expressed directionally.
- If not, then a two-tailed test would be required as foll:

 $H_0: \pi = 0.40$

*H*₁: $\pi \neq 0.40$

Step 2: Select an Appropriate Test

- The **test statistic** measures how close the sample has come to the null hypothesis.
- The test statistic often follows a well-known distribution (eg, normal, *t*, or chi-square).
- In our example, the *z* statistic, which follows the standard normal distribution, would be appropriate.

$$z = \frac{p - \pi}{\sigma_p}$$

Where σ_p is standard deviation

Step 3: Choose Level of Significance

Type I Error

- Occurs if the null hypothesis is rejected when it is in fact true.
- The probability of type I error (α) is also called the level of significance.

Type II Error

- Occurs if the null hypothesis is not rejected when it is in fact false.
- The probability of type II error is denoted by $\boldsymbol{\beta}$.
- Unlike α , which is specified by the researcher, the magnitude of β depends on the actual value of the population parameter (proportion).

It is necessary to balance the two types of errors.

Step 3: Choose Level of Significance

Power of a Test

- The power of a test is the probability (1 - β) of rejecting the null hypothesis when it is false and should be rejected.
- Although β is unknown, it is related to α. An extremely low value of α (e.g., = 0.001) will result in intolerably high β errors.



Step 4: Collect Data and Calculate Test Statistic

- The required data are collected and the value of the test statistic computed.
- In our example, 30 people were surveyed and 17 shopped on the internet. The value of the sample proportion is p = 17/30 = 0.567.
- The value of σ_p is:

 $\sigma_{p} = 0.089$

Step 4: Collect Data and Calculate Test Statistic

The test statistic *z* can be calculated as follows:

$$z_{CAL} = \frac{\hat{p} - \pi}{\sigma_p}$$

= <u>0.567-0.40</u> 0.089

= 1.88

Step 5: Determine Probability Value/ Critical Value

- Using standard normal tables (Table 2 of the Statistical Appendix), the area to the right of z_{CAL} is .0301 (z_{CAL} =1.88)
- The shaded area between 0 and 1.88 is 0.4699. Therefore, the area to the right of 1.88 is 0.5 0.4699 = 0.0301.
- Thus, the p-value is .0301
- Alternatively, the critical value of *z*, called z_{α} , which will give an area to the right side of the critical value of α =0.05, is between 1.64 and 1.65. Thus z_{α} =1.645.
- Note, in determining the critical value of the test statistic, the area to the right of the critical value is either α or α/2. It is α for a one-tail test and α/2 for a two-tail test.

Steps 6 & 7: Compare Prob and Make the Decision

- If the prob associated with the calculated value of the test statistic (z_{CAL}) is <u>less than</u> the level of significance (α), the null hypothesis is rejected.
- In our case, the p-value is 0.0301. This is less than the level of significance of α =0.05. Hence, the null hypothesis is rejected.
- Alternatively, if the calculated value of the test statistic is greater than the critical value of the test statistic (z_{α}), the null hypothesis is rejected.

Steps 6 & 7: Compare Prob and Make the Decision

- The calculated value of the test statistic z_{CAL} = 1.88 lies in the rejection region, beyond the value of z_{α} =1.645. Again, the same conclusion to reject the null hypothesis is reached.
- Note that the two ways of testing the null hypothesis are equivalent but mathematically opposite in the direction of comparison.
- Writing Test-Statistic as TS:

If the probability of $TS_{CAL} < significance level (<math>\alpha$) then reject H₀ but if $TS_{CAL} > TS_{CR}$ then reject H₀.

Step 8: Mkt Research Conclusion

• The conclusion reached by hypothesis testing must be expressed in terms of the marketing research problem.

 In our example, we conclude that there is evidence that the proportion of Internet users who shop via the Internet is significantly greater than 0.40. Hence, the department store should introduce the new Internet shopping service.

Using a t-Test

- Assume that the random variable X is normally dist, with unknown pop variance estimated by the sample variance s².
- Then a **t test** is appropriate.

- The t-statistic, $t = (\overline{X} \mu)/s_{\overline{X}}$ is t distributed with n 1 df.
- The t dist is similar to the normal distribution: bell-shaped and symmetric. As the number of df increases, the t dist approaches the normal dist.



Hypothesis Testing for Differences



Two Independent Samples: Means

• In the case of means for two independent samples, the hypotheses take the following form.

 $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$

- The two populations are sampled and the means and variances computed based on samples of sizes *n*1 and *n*2.
- The idea behind the test is similar to the test for a single mean, though the formula for standard error is different
- Suppose we want to determine if internet usage is different for males than for females, using data in Table 15.1

Internet Usage Data

Table 15.1

to an artefuti	Der	Familiority	boter net	Attitude	Toward	Usags of	Internet
TERMINE THEFT	7915NA	1 CREATING AV	Usage	internet	Technology	Shopping	Santing
Sal and actual i	1 50	7.30	14 00	7.00	6.00	1.00	1.00
	3 516	2 33	2 00	3.00	3.00	2.00	2.00
2	2.90	2 143	3 00	4.60	3.00	÷90	2.00
÷	2.00	3.00	3.60	7.00	5.00	1.00	2.00
4	2.00	7.00	13 00	7.05	7.00	1.00	1.00
5	2.00	7.00	6.00	5 03	4.00	1 60	2.00
8	2.00	200	2.00	4 05	5.07	2.60	Z.00
6	2.00	2.00	6.00	5.00	4 03	2.60	2.60
8	2.00	3.00	0.00	8.00	A CO	1.00	2.00
9	2.00	3.00	15.00	7 00	5.00	1.92	2.00
10	1.00	9.00	200	4.00	5.00	2 00	2.00
11	2.00	4.00	0.00	3.00	5.00	2.00	2.00
12	2.00	5,00	4.00	3.00	5.00	2 00	1 00
13	1.00	6.00	8.00	3.00	2.00	2.06	2.00
:4	1.00	6.00	5.00	503	6.00	1.00	2.00
16	1.00	8.00	0.00	3.UJ 4.0D	2.00	2.00	2.00
* 6	2.00	4.40	2.60	9.00 5.00	3.00	1 00	1.00
17	1.09	6.90	2 UU	5.00	4.00	1 00	2.00
18	1.00	4 30	4 60	3.00	\$ 00	1.00	1.00
16	1.00	7.90	14.00	0.00	A 00	2.00	2.00
20	2.00	5.00	6 00	0.00	2.00	2 30	2.09
<u>2</u> -i	1.00	6.00	9.0U	4,00	2.00 A DC	2 30	1.00
22	1.00	5.60	5.00	5.00	7.00	2 0.3	2.00
23	2.00	3.00	2.00	4.00	2.00	4 20	1.00.
24	1.00	7.00	15.00	5.00	0.00	1.60	2 00
25	2.00	6.00	6.00	5.00	3.00	4.50	1.00
28	1.00	6.00	13.00	6.00	0.00	4.00	1.00
2.7	2.00	5.00	4.00	5.00		1.00	2 00
23	2.00	4.00	2.03	3.02	2.00	4.05	200
29	<.00	4.00	4.00	5.99	3,00	1.00	2.00
30	1.DQ	3.00	3.00	7.00	5.60	1.00	

Two Independent-Samples: *t* Tests Table



Two Independent Samples: Proportions

- Consider data of Table 15.1
- •Is the proportion of respondents using the Internet for shopping the same for males and females? The null and alternative hypotheses are:

*H*₀:
$$\pi_1 = \pi_2$$

*H*₁: $\pi_1 \neq \pi_2$

•The test statistic is similar to the one for difference of means, with a different formula for standard error.

Summary of Hypothesis Tests for Differences



Summary of Hypothesis Tests for Differences

Two Indep Samples	Application	Scaling	Test/Comments
Two indep samples	Means	Metric	Two-group <i>t</i> test
		Matric	F test for equality of variances
Two indep samples	Proportions	Nonmetric	Ztest Chi -square test