TORSIONAL DEFLECTION

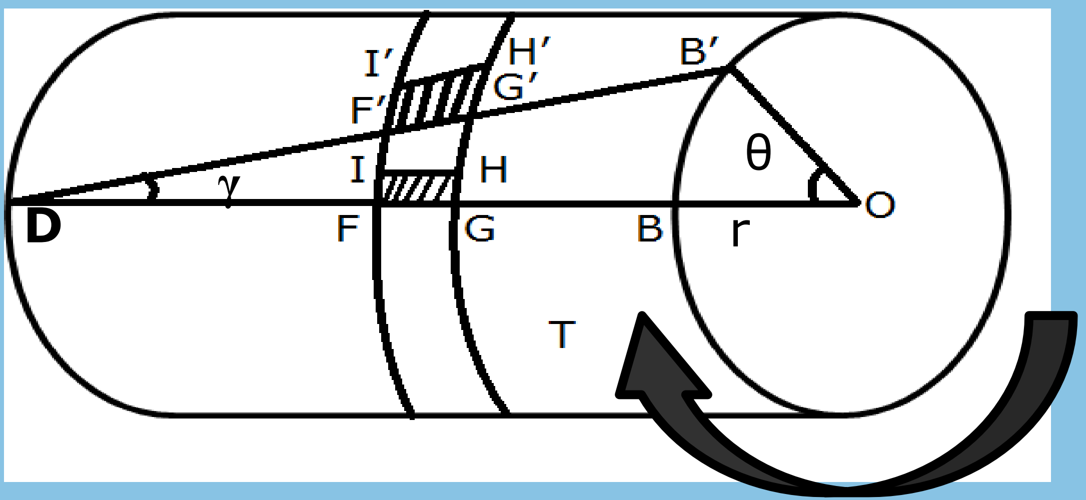
What is torsion ?

- **Torsion** is the twisting of an object due to an applied torque.
- Torsional deflection is the angular displacement or deformation of a body when a twisting force is applied.

Derivation for torsional deflection of a circular shaft

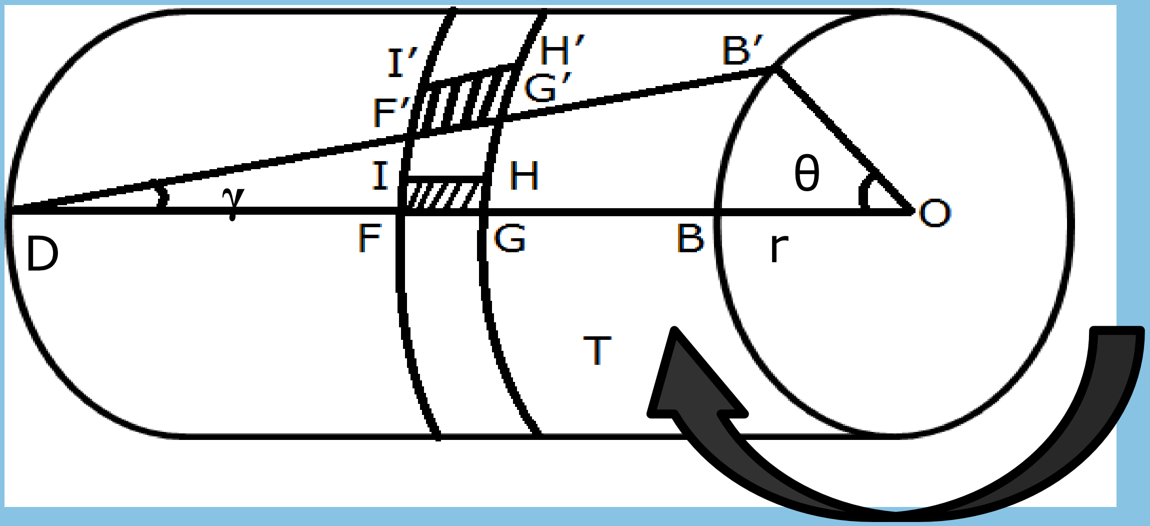
Assumptions

1)The material of the shaft should be homogenous and isotropic.



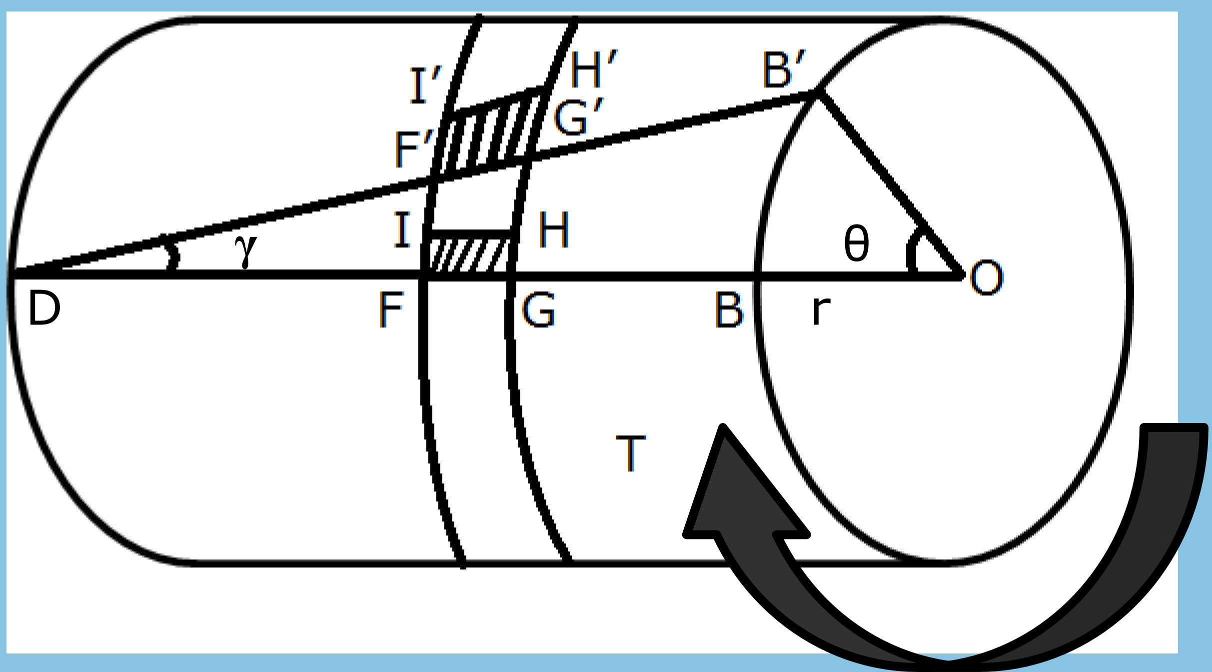
Assumptions

2)The cross section perpendicular to the longitudinal axis of the shaft remains perpendicular to the shaft even after the application of torque



Assumptions

3) Surface elements of the cylinder remain straight even after twisting takes place.



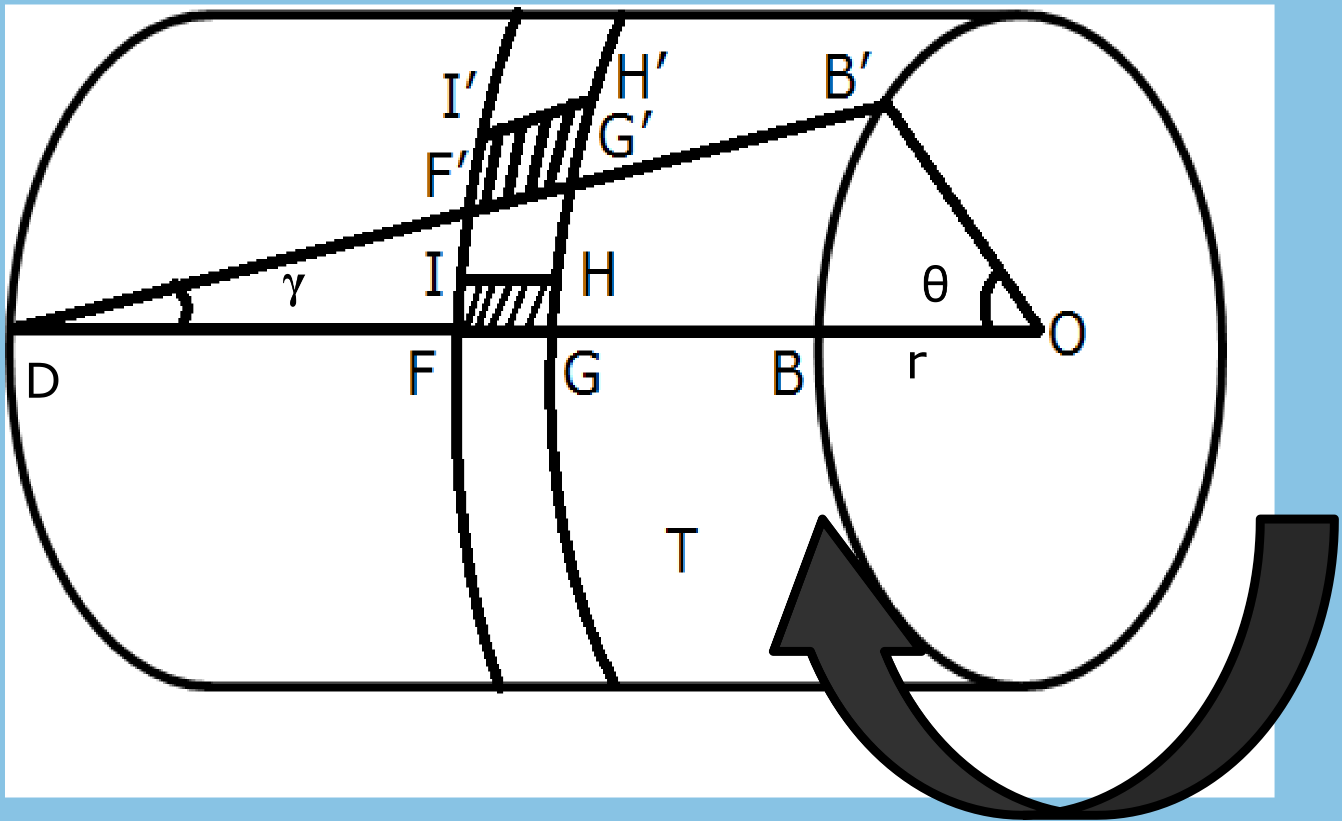
Assumptions

4)A typical rectangle such as IFGH will take the shape of a parallelogram I′F′G′H′ after twisting.The amount of angular distortion is given by angle γ which represents shear strain.

Thus,

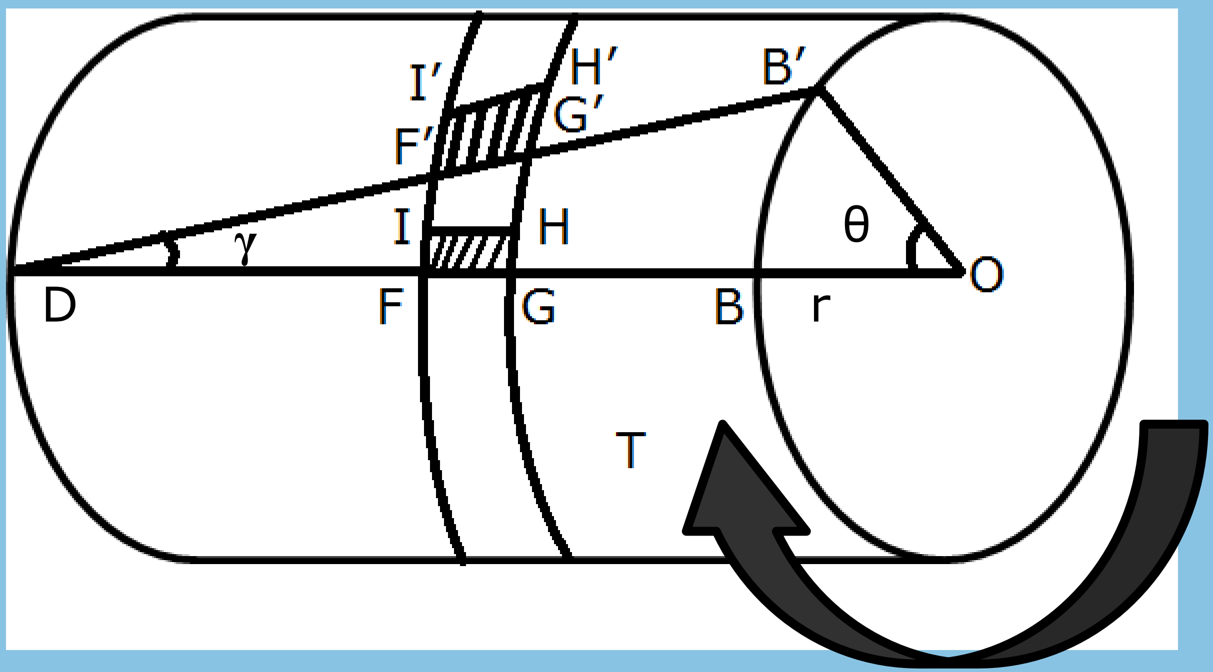
$$\gamma=r\theta/L$$

Assumptions



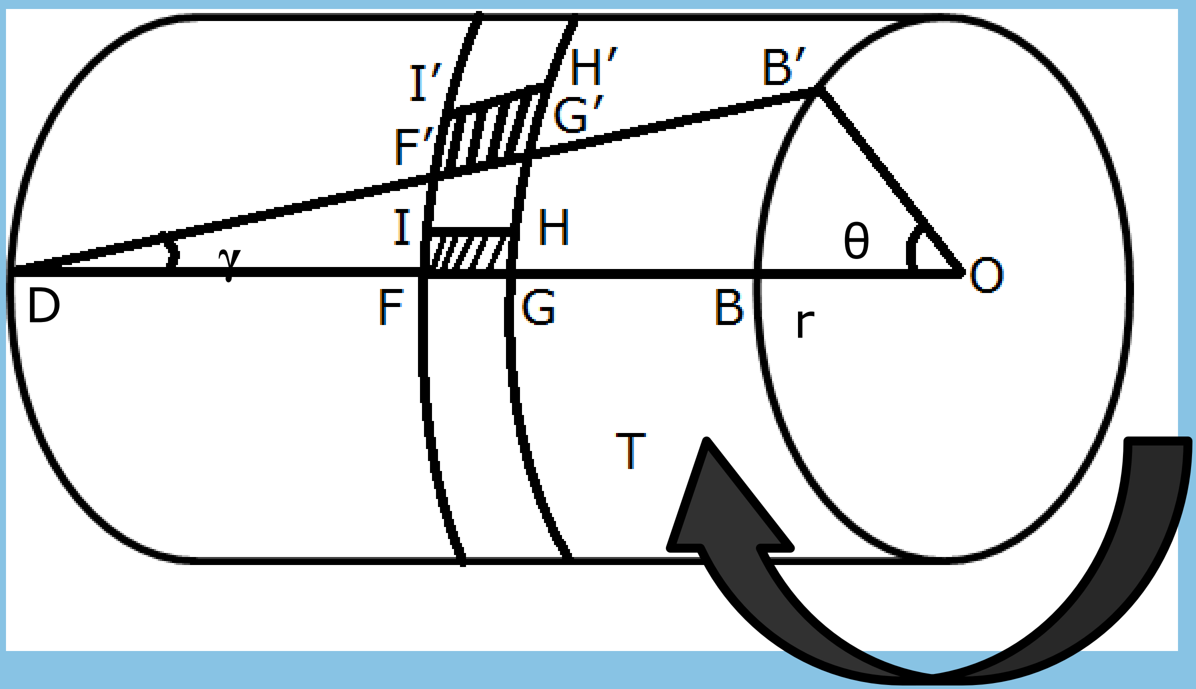
Assumptions

5) Length of longitudinal elements remain constant under action of external torque.



Assumptions

6) Hooke’s Law relates shear stress to shear strain.



Derivation

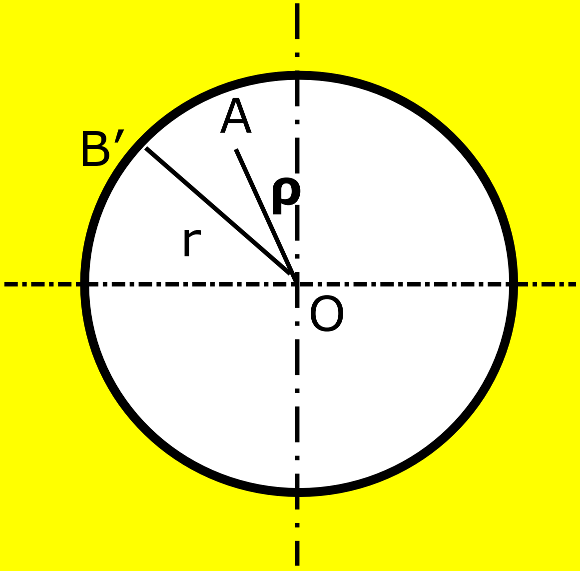
Let us consider that the shaft is subjected to shear stress. Now if we consider any radial position , ρ of an element OA of the shaft ,then by assumption 4, we have

$$\gamma = \gamma_\rho = \rho\theta/L$$

As $\rho = r$ *at the periphery,*

$$\gamma = \gamma_r = r\theta/L$$

So , $\gamma_\rho/\gamma_r = \rho/r$ *....(1)*



Derivation

From assumption 6,

$\tau = G\,\gamma$ *where* τ *is Shear stress ,*γ *is Shear strain and G is modulus of rigidity.*

$\tau_\rho \,/\, \tau_r = \gamma_\rho \,/\, \gamma_r$ *......(2)*

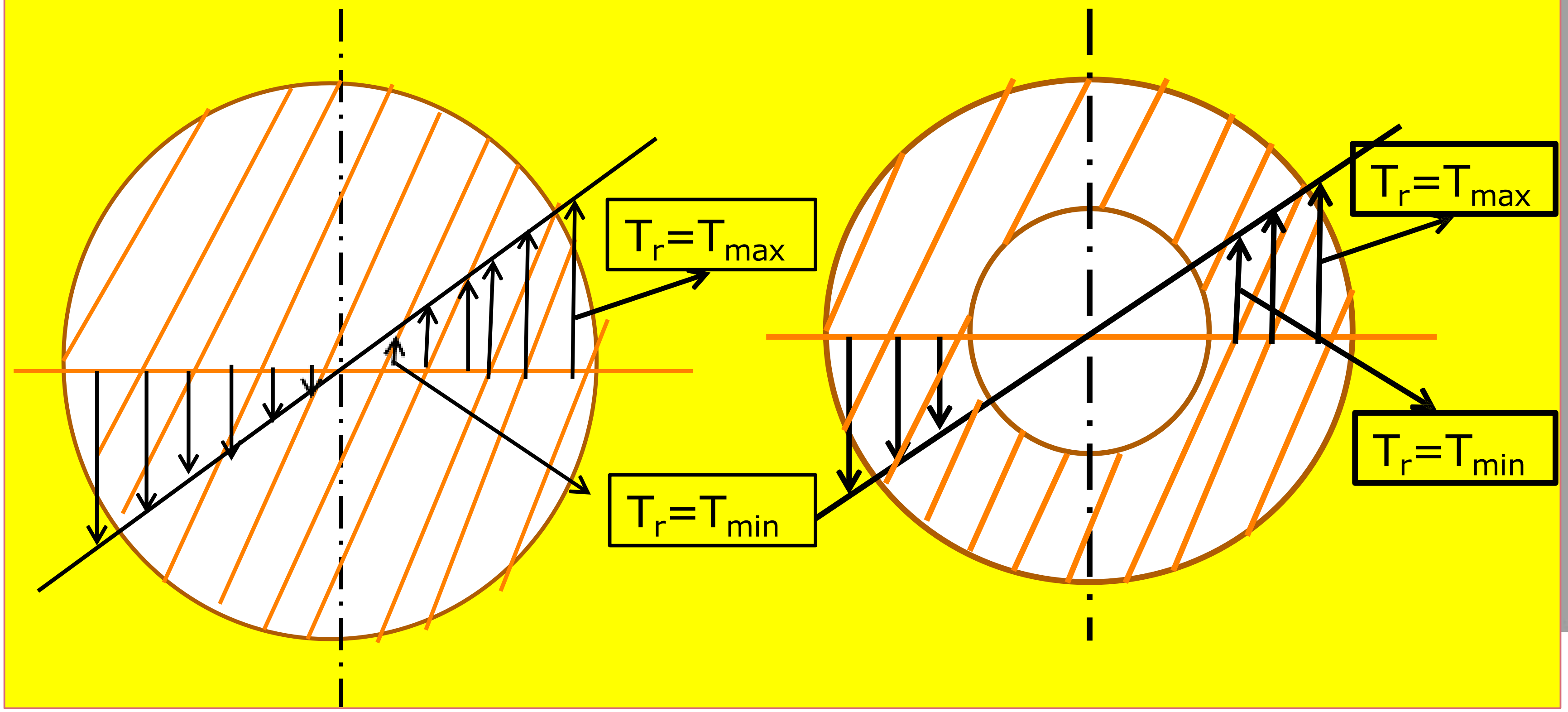
Where τ_p *is shear stress at point A on shaft Cross section.*

From (1) and (2),

$\tau_\rho \,/\, \tau_r = \rho / r$ *.....(3)*

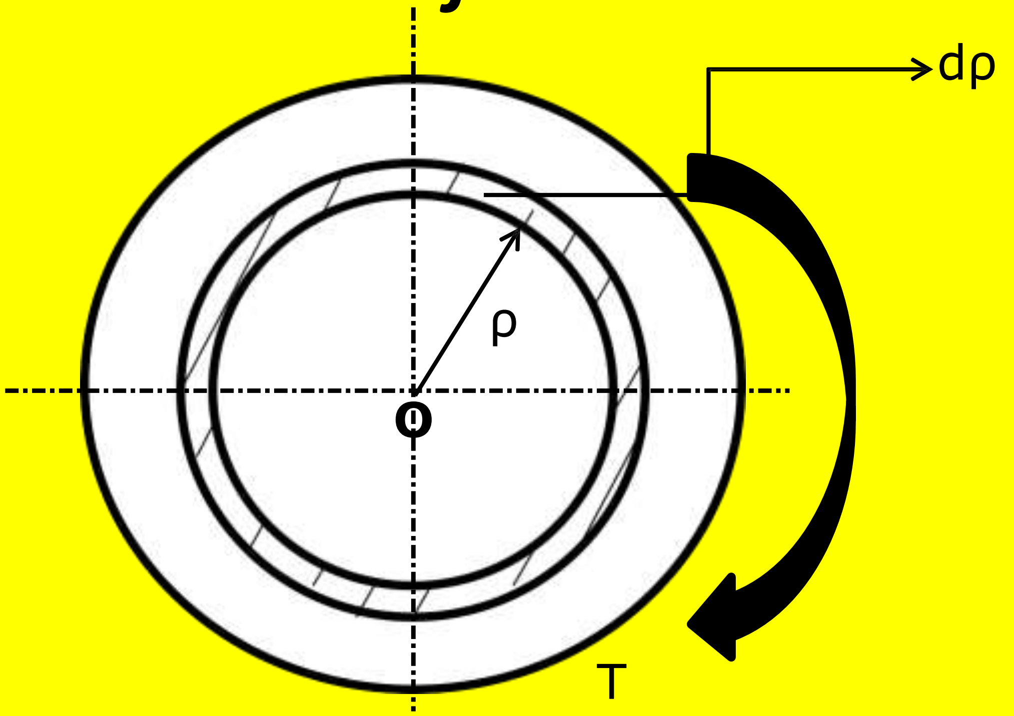
Derivation

Thus shear stress varies linearly with radial distance.



Derivation

Now to correlate shear stress with torque T let us consider a typical cross section of the solid circular shaft.



Derivation

If we focus on annular differential area we get

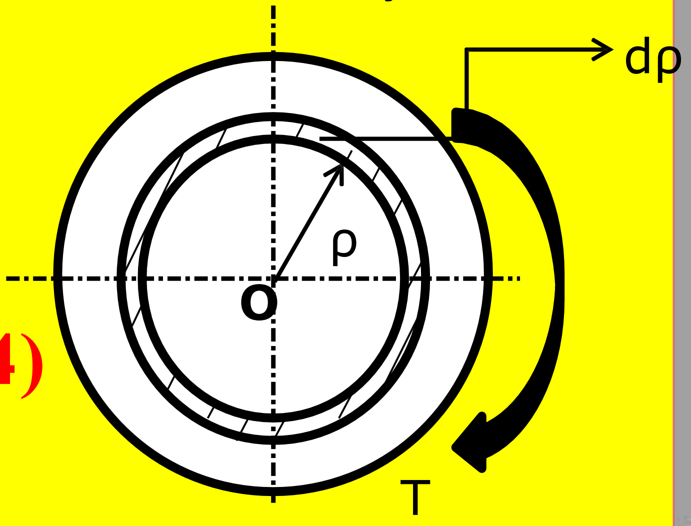
$dA = 2\pi\rho d\rho$

If shear stress caused by external torque T on this elemental strip is τ_ρ then differential force acting on area dA is dF, which is given by,

$dF = \tau_\rho\, dA = \tau_\rho \times 2\pi\rho d\rho$

Differential torque dT is

$dT = \rho dF = \rho \times \tau_\rho \times 2\pi\rho d\rho \quad ....(4)$



Derivation

Putting τ_ρ from equation (3)

$$dT = 2\pi\rho^3\tau_r/r\ d\rho = (\tau_r/r)2\pi\rho^3\ d\rho$$

Integrating both sides,

$$\int_0^T dT = 2\pi\ (\tau_r/r)\int_0^r \rho^3\ d\rho$$

$$T = 2\pi(\tau_r/r)\ X\ (r^4/4) = \tau_r/r(\pi r^4/2) = \tau_r J/r$$

$J = \pi r^4/2$ = Polar moment of inertia of circular cross sectional area

Thus, $T = \tau_r J/r$ *and so* $\tau_r = Tr/J$ *....(5)*

This is the desired relation between shear stress and external torque.

Derivation

We shall now find relation between angle of twist θ and applied torque. By hooke's law, $\tau_r = G\gamma_r$

So $\tau_r = Gr\theta/L$ *....(6)*

From (5) and (6),

$Tr/J = Gr\theta/L$

Rearranging, $\theta = TL/GJ$ *...(7)*

$T/J = G\theta/L = \tau_r/r$ *....(8)*

This is the required Torsion Equation for a circular shaft.

Derivation

On putting J=$\pi(d^4)/32$ and $J=\pi(d_2^4 - d_1^4)/32$ *in the torsion equation for solid and hollow shaft respectively ,we get*

1) $\theta = TL/(G\,\pi(d^4)/32)$ *[FOR SOLID SHAFT]*

2) $\theta = TL/(G\,\pi(d_2^4 - d_1^4)/32)$

[FOR HOLLOW SHAFT]

θ *is the torsional deflection or angle of twist*

CONCLUSION

- Measurement of torsional defection helps in establishment of proper dimensions for shaft and other parts so that failure is prevented.