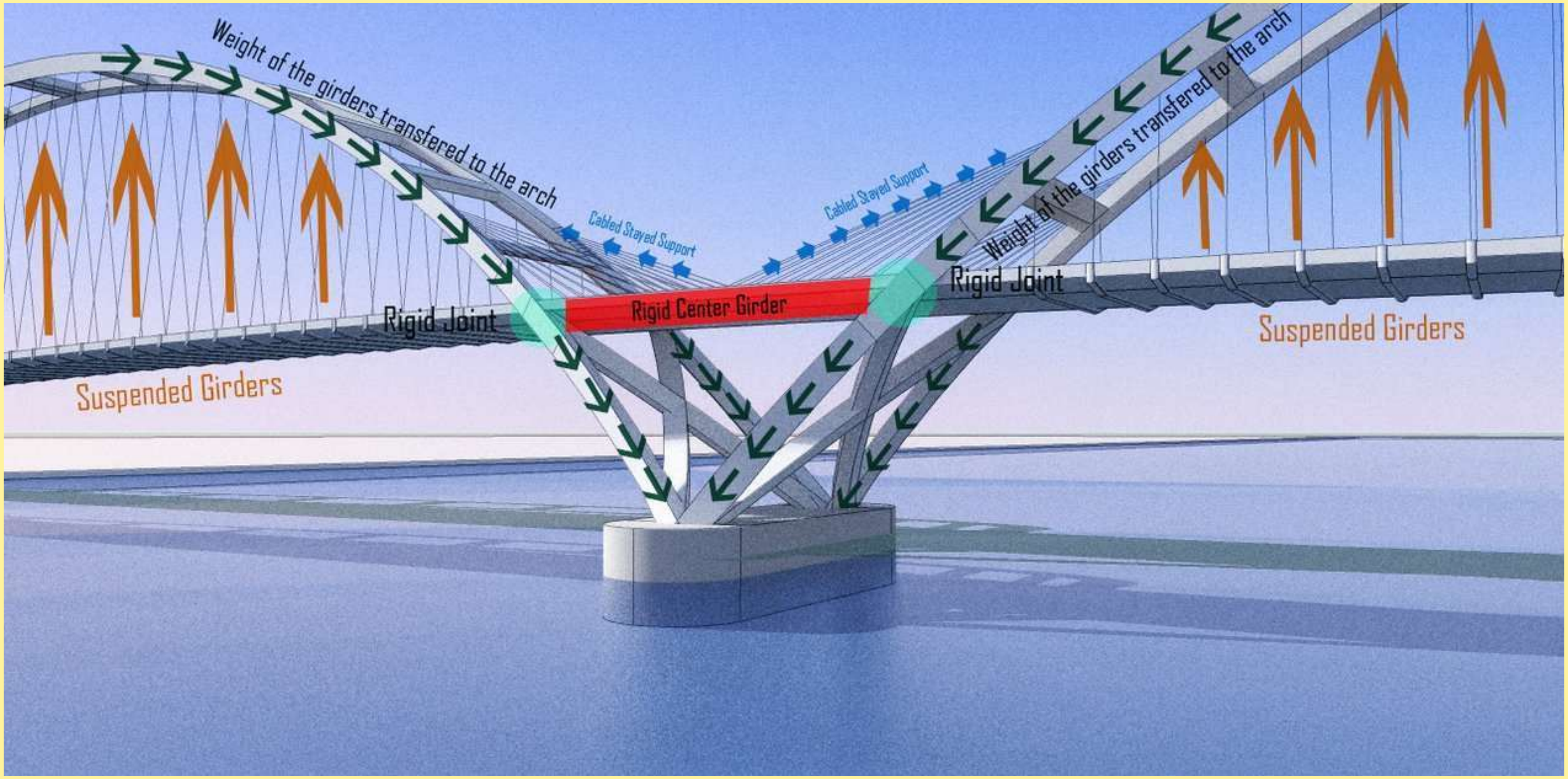
SHEAR FORCE AND BENDING MOMENT



SHEAR FORCE AND BENDING MOMENT

❖**Beam: It is a structural member which is subjected to transverse load.**

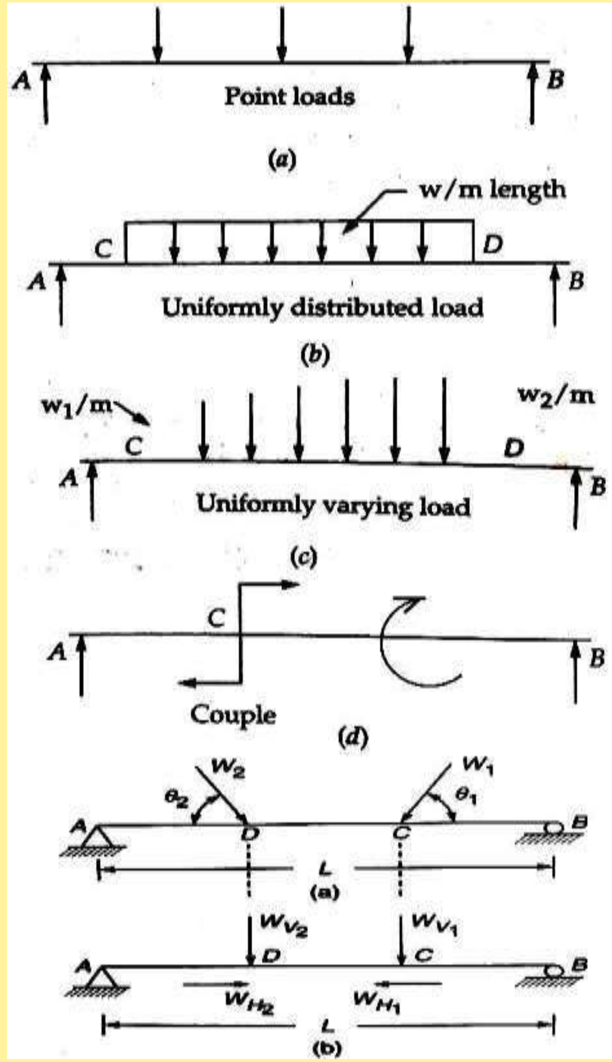
❖**Transverse load: The load which is perpendicular or it has non zero component perpendicular to longitudinal axis of beam.**

❖**Shear force: it is the unbalanced force on either side of a section parallel to cross section.**

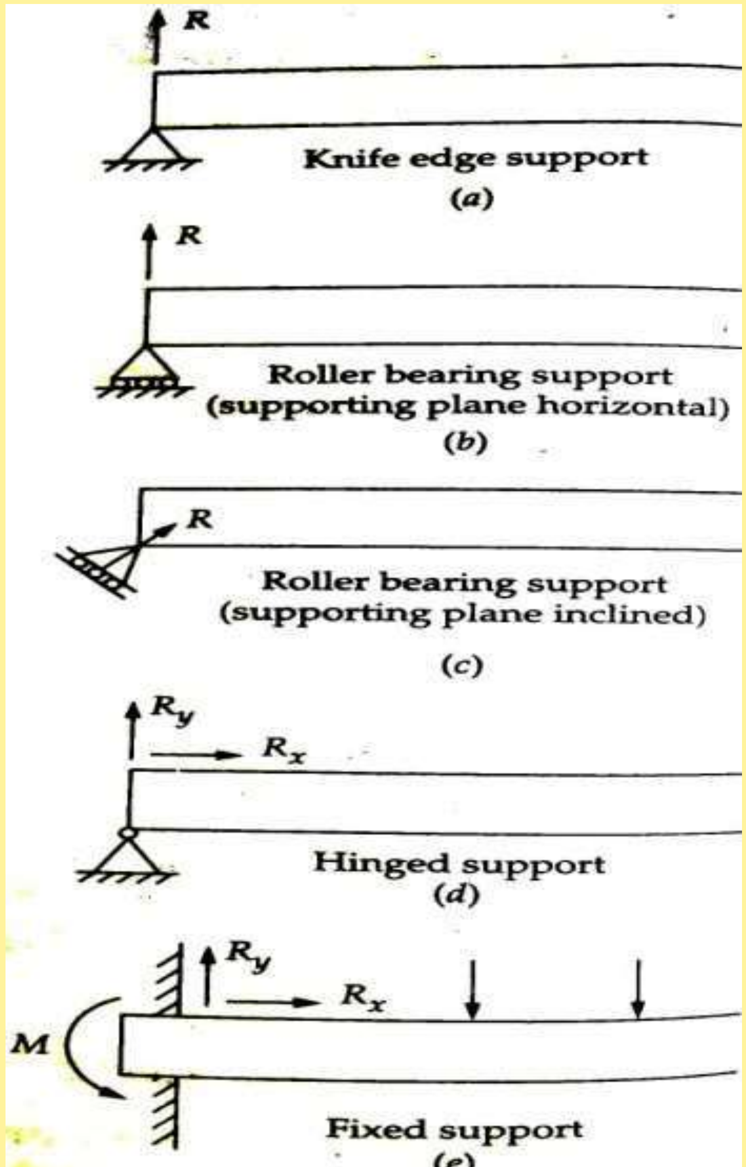
❖**Moment: It is the product of force and perpendicular distance between line of action of the force and the point about which moment is required to be calculated.**

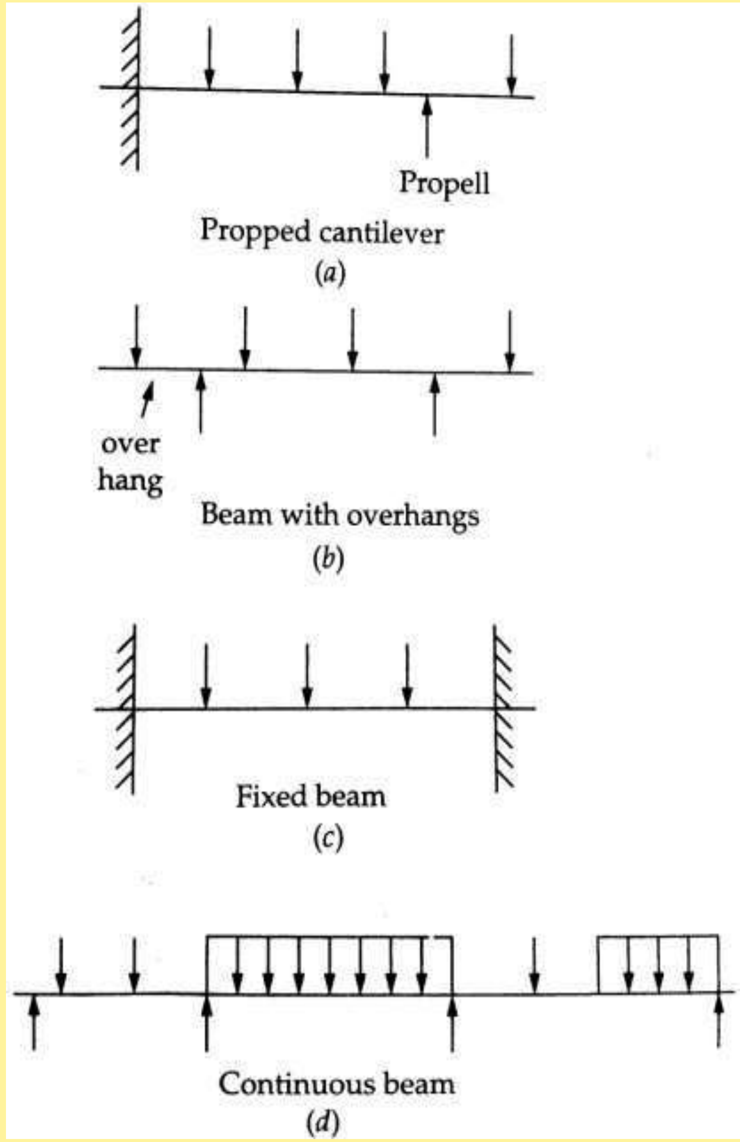
❖**Bending moment: It is the unbalanced moment on either side of section, in the plane of beam.**

TYPES OF LOAD

Sr. No.	Type of load	Example	Description
1.	Point Load		Concentrated Load acts at a point.
2.	Uniformly Distributed Load		UDL:- uniform load distribution over wide area. Rate of loading per unit length.
3.	Uniformly Varying Load (Triangular Distributed Load)		UVL:- Intensity of load at one point to that at the other, eg. w_1/m at C to w_2/m at D
4.	Couple		A beam may be subjected to a couple.
5.	Oblique Load		The effect of Horizontal components is to cause a thrust in the beam. Vertical components of the load cause bending and shear & are treated as usual vertical loads on a beam

TYPES OF SUPPORTS

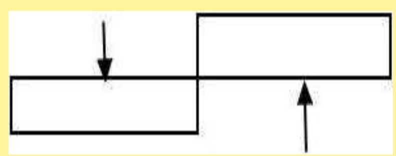
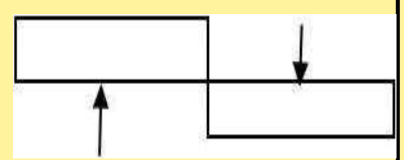
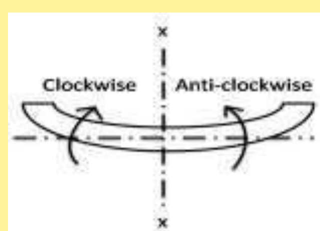
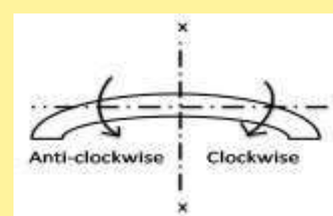
Sr. No.	Type of support	Example	Description
1.	Knife Edge Support		Contact Area Insignificant. Provides only vertical reaction No resistance to turning or lateral displacement
2.	Roller Support (Horizontal Plane)		Rollers on Horizontal Plane Support reaction is vertical. No resistance to turning or lateral displacement
3.	Roller Support (Inclined Plane)		Rollers on Inclined Plane Support reaction is perpendicular to inclined plane. Allow turning or lateral displacement.
4.	Hinged/Pin Support		Allows turning but doesn't allow any lateral movement. Support reaction could be in any direction. Can be determined by resolving applied in horizontal. & Vertical. direction
5.	Fixed Support		Doesn't allow rotation or translation.

TYPES OF BEAMS			
Sr. No.	**Type of Beam**	**Example**	**Description**
1.	Cantilever Beam		Beams have one end rigidly built into the support. Large span or heavy loads provided by additional support are known as propel and beam as a propped cantilever.
2.	Simply Supported Beam		Beams with knife edge supports or roller supports at ends.
3.	Beams with Overhangs		Portion of a beam that goes beyond the support is called overhanging, may be on one or both ends.
4.	Fixed Beams		Rigidly built-in-supports at both ends. Beam have support reaction and a fixing moments at each end.
5.	Continuous Beams		Beams that cover more than one span.

BEAMS IN BENDING

Both shear force and bending moment are vector quantities requiring a convention of signs in order that values of opposite sense may be separated. Mathematical signs are chosen since it is in calculation problems that it becomes necessary to use such a convention

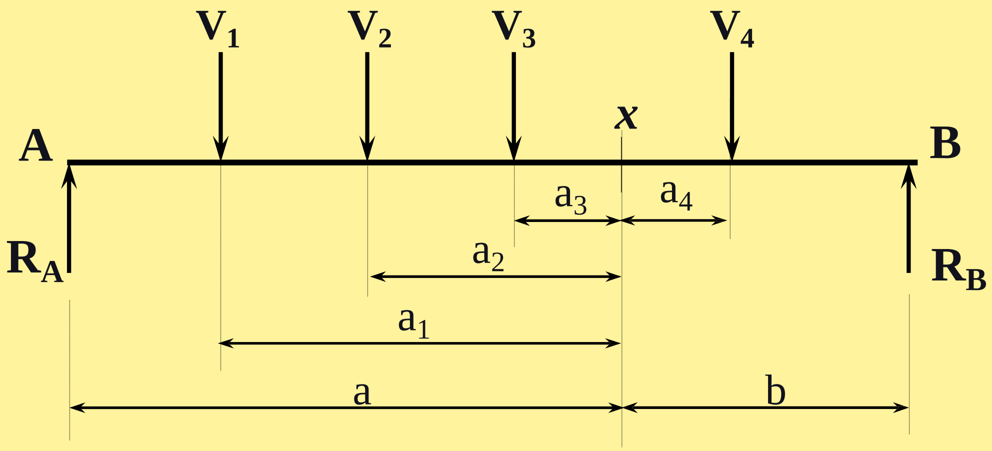
TABEL 1. Sign convention and units for shearing force and bending moment

LOAD EFFECT	**SYMBOL**	**SIGN CONVENTION**		**UNITS**
		POSITIVE (+)	**NEGATIVE (-)**	
Shear force	**Q, V, S**			N KN
Bending moment	**M**	 (bottom fibers in tension)	 (top fibers in tension)	N·mm kN·m

BEAMS IN BENDING

The *shearing force*, at any transverse section in a loaded beam, is the algebraic sum of all the forces acting on one (either) side of the section.

The *bending moment*, at any transverse section in a loaded beam, is the algebraic sum of the moments about the sections of all the forces acting on one (either) side of the section.



Working to the left of *x*:

$$Q_x = R_A - V_1 - V_2 - V_3$$

$$M_x = R_A \cdot a - V_1 \cdot a_1 - V_2 \cdot a_2 - V_3 \cdot a_3$$

Working to the right of *x*:

$$Q_x = -R_B + V_4$$

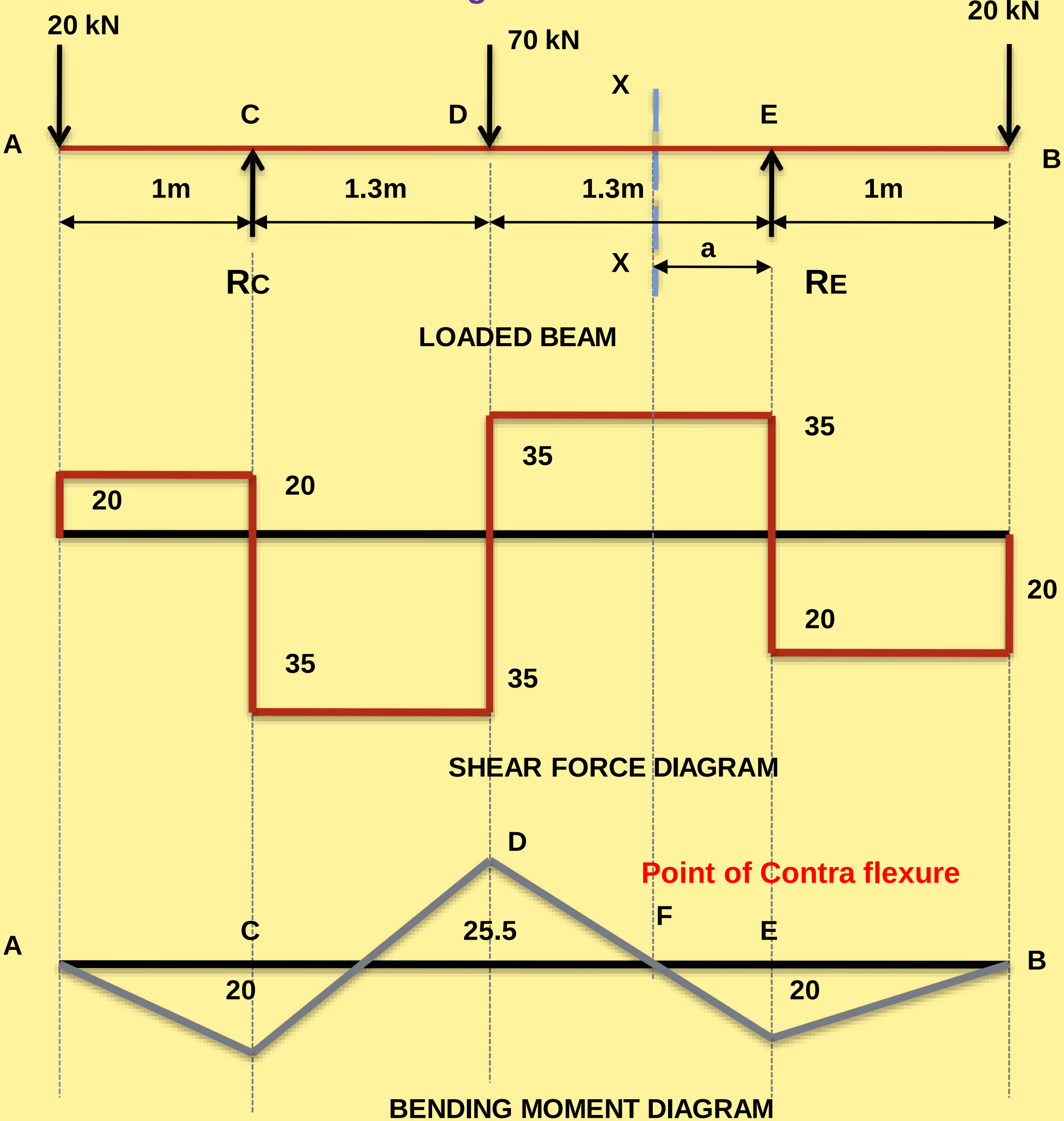
$$M_x = R_B \cdot b - V_4 \cdot a_4$$

THINGS TO REMEMBER FOR DRAWING OF S.F & B.M

- ❖ Start from right hand section.
- ❖ Use Sign convention of the side which you are choosing i.e Right or Left.
- ❖ If the thing are complicated use other side of section.
- ❖ Start from zero and end to zero. B.M at the ends will be zero.
- ❖ End point of S.F. will be equal and opposite to the reaction at that point.
- ❖ Mark the points and draw the diagram considering the type of load.
- ❖ At change in nature of forces there will be two points in shear force diagram.
- ❖ At Couple there will be two points in B.M Diagram.

- Beams are assumed to be always straight, horizontal & of uniform c/s & structure, unless otherwise specified.
- Self weight of Beam neglected unless its definite value is given.
- ▪S.F may be max. at supports or under point loads or where S.F. is zero where B.M. may be maximum at point where S.F. is zero or where S.F. changes its nature.
- Where the B.M changes its nature is known as Point of Contra Flexure or Point of Inflexion.

Draw S.F. and B.M. diagrams for the loaded Beam Point Load



Calculations

Symmetrical loading

$$R_C = R_E = \frac{20 + 70 + 20}{2} = 55 \text{ kN}$$

Shear Force

S.F at B = -20kN
S.F at E = 55 - 20 = 35 kN
S.F at D = -20 – 70 + 55 = -35 kN
S.F at C = -20 + 55 - 70 + 55 = 20 kN

Bending Moment

BM_E = -20 X 1 = -20 kNm.

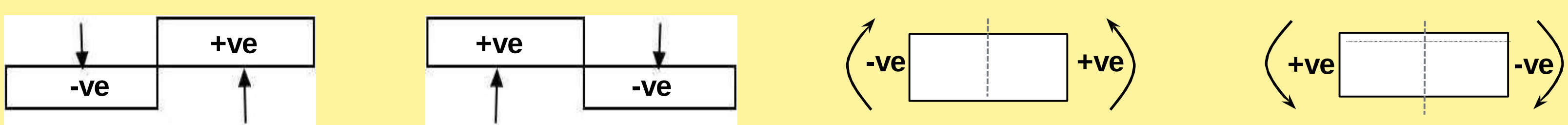
BM_D = -20 X 2.3 + 55 X 1.3 = 25.5kNm.

BM_C = -20(1.3 + 1.3 +1) + 55(1.3 + 1.3) -70 X 1.3 = -20 kNm.

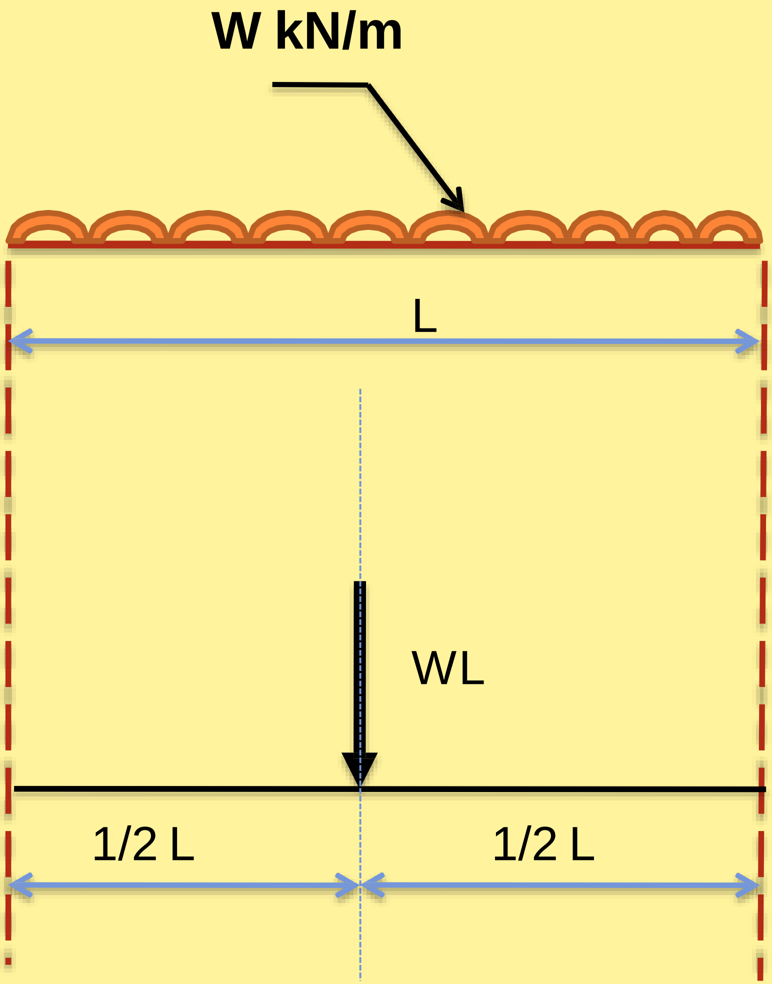
BM_X = - 20 (a + 1) + 55a = 35 a – 20

Point of Contra flexure BM_X = 0

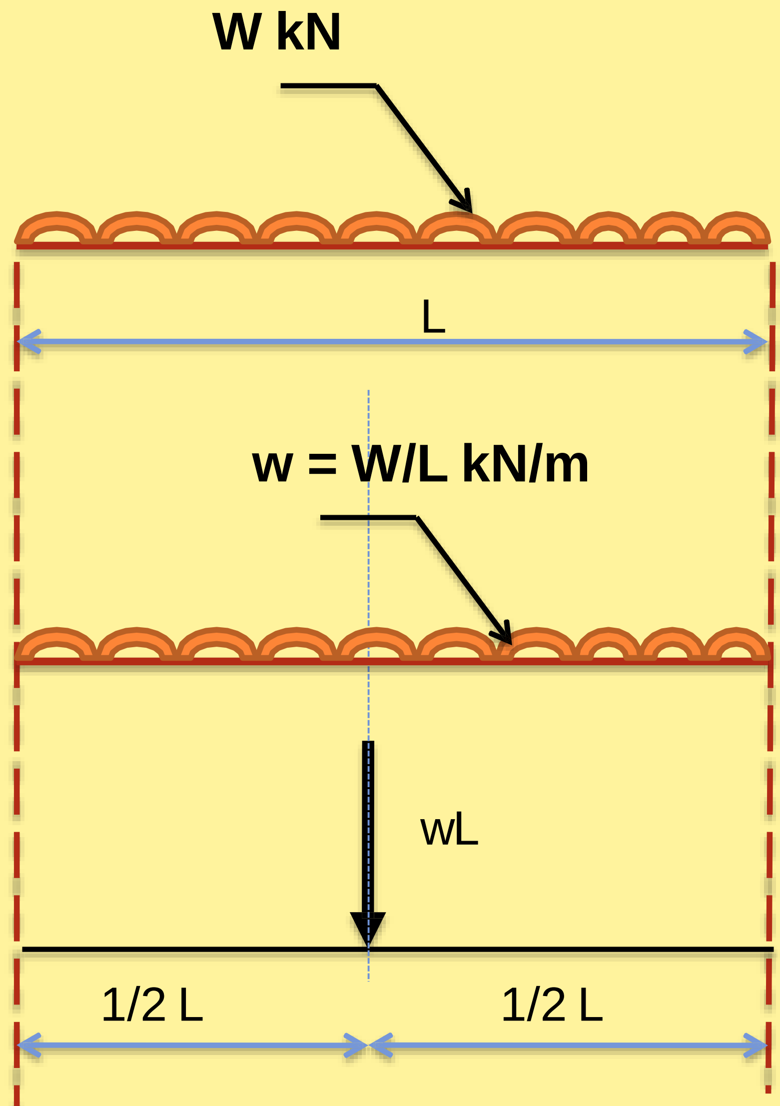
a = 0.57



UNIFORMLY DISTRIBUTED LOAD

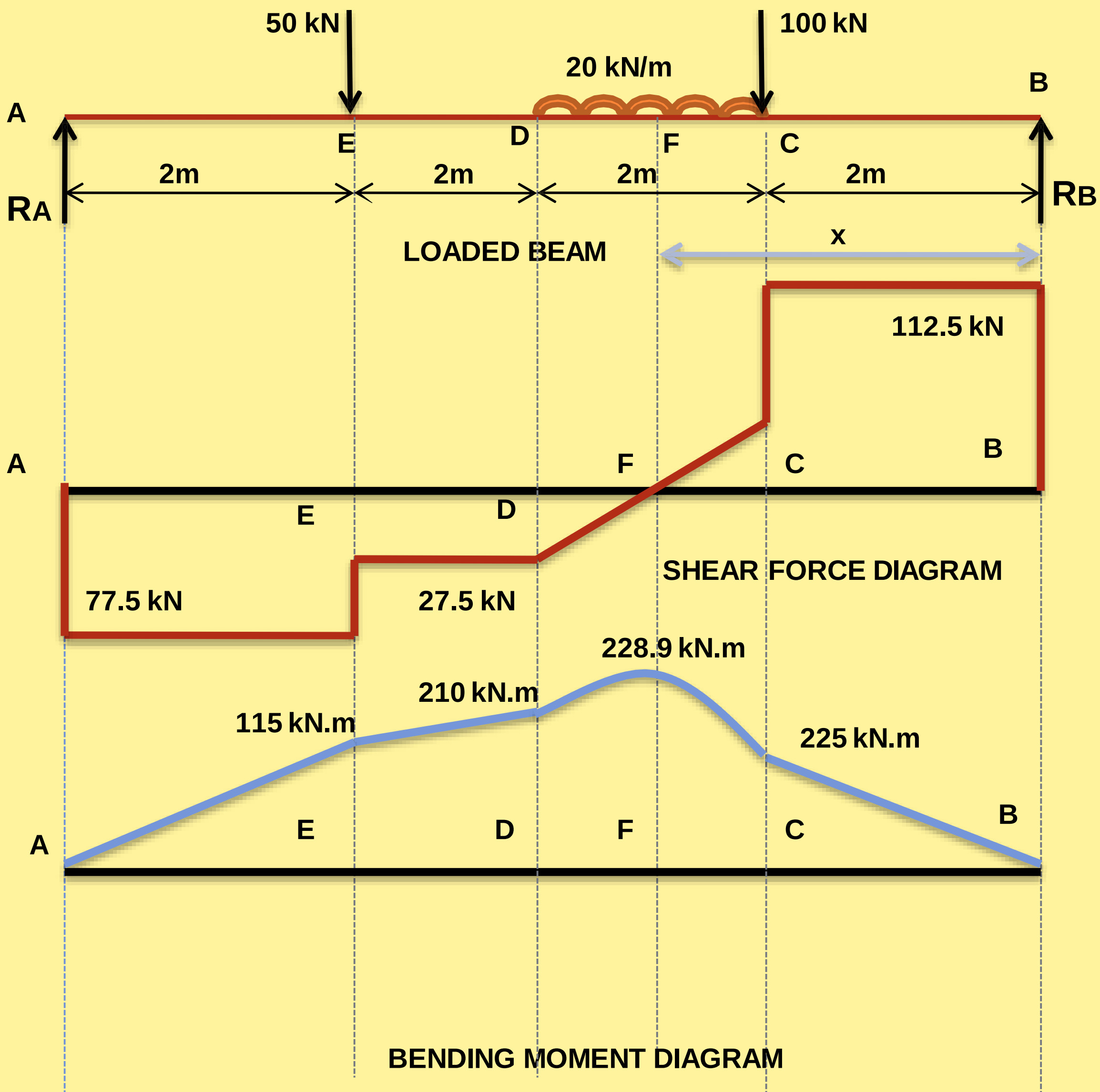


Load act centre of UDL



There will be Parabola in B.M. and inclined line in S.F. diagram

EFFECT OF UNIFORMLY DISTRIBUTED LOAD



Calculations

Reactions

$R_A + R_B = 100 = 20 \times 2 + 50 = 190$

M_A,

$R_B \times 8 = 100 \times 6 + 20 \times 2 \times 50 \times 2$

$R_B = 112.5$ kN, RA = 77.5 kN

Shear Force

S.F at B = 112.5 kN (+ ve)

S.F at C = 112.5 -100 = 12.5 kN

S.F at D = 112.5 -100 – 20 x 2 = -27.5 kN

S.F at E = 112.5 – 100 – 20 X 2 – 50 = -77.5 kN

S.F. at A = – 77.5 kN

S.F. at F = 0

S.F. at F = 112.5 – 100 – 20 (x – 2)

X = 2.625 m

Bending Moment

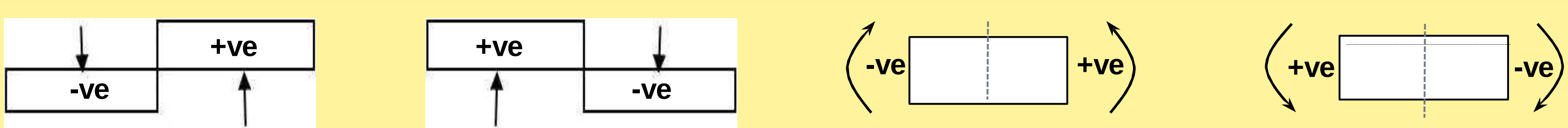
BM_A = 0 , **BM_B = 0**

BM_C = 112.5 X 2 = 225 kN.

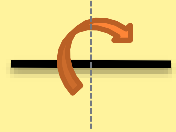
BM_D = 112.5 X 4 – 100 X 2 – 20 X 2 X 2/2
= 210 kN.m

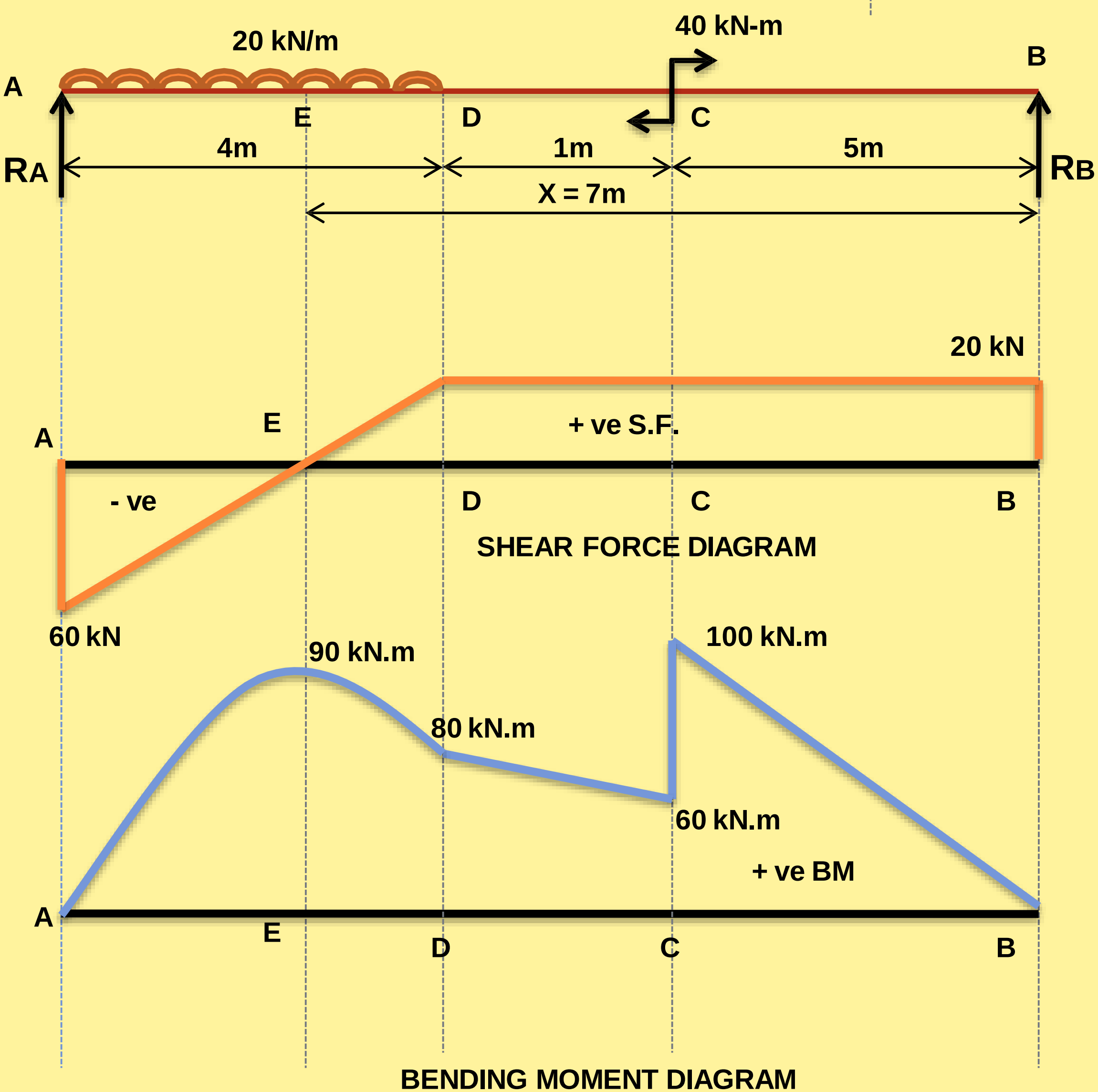
BM_E = 112.5 X 6 – 100 X 4 – 20 X 2 X 3
= 155 kN.m

BM_F = 112.5 X 2.625 – 100 X 0.625 – 20 X 0.625 X 0.625/2 = 228.9 kN.m



EFFECT OF COUPLE





Calculations

Reactions

$R_A + R_B = 20 \times 4 = 80$

M_A,

$R_B \times 10 = 40 + 20 \times 4 \times 2$

$R_B = 20$ kN, $R_A = 60$ kN

Shear Force

S.F at B = 20 kN

S.F at C = 20 kN

S.F at A = -60 kN

S.F at E = 0

S.F at E = 20 – 20 (x-6)

x = 7m

Bending Moment

$BM_A = 0$, $BM_B = 0$,

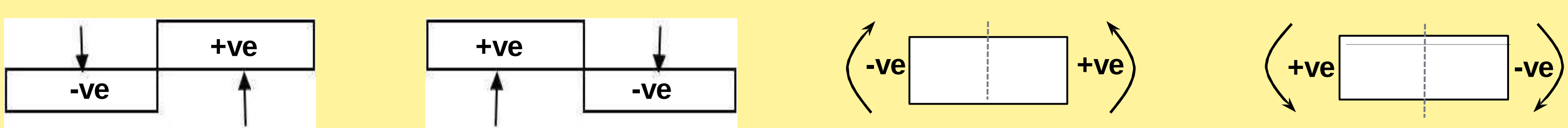
$BM_C = 20 \times 5 = 100$ kN.m (just before)

$BM_C = 100 - 40 = 60$ (just after)

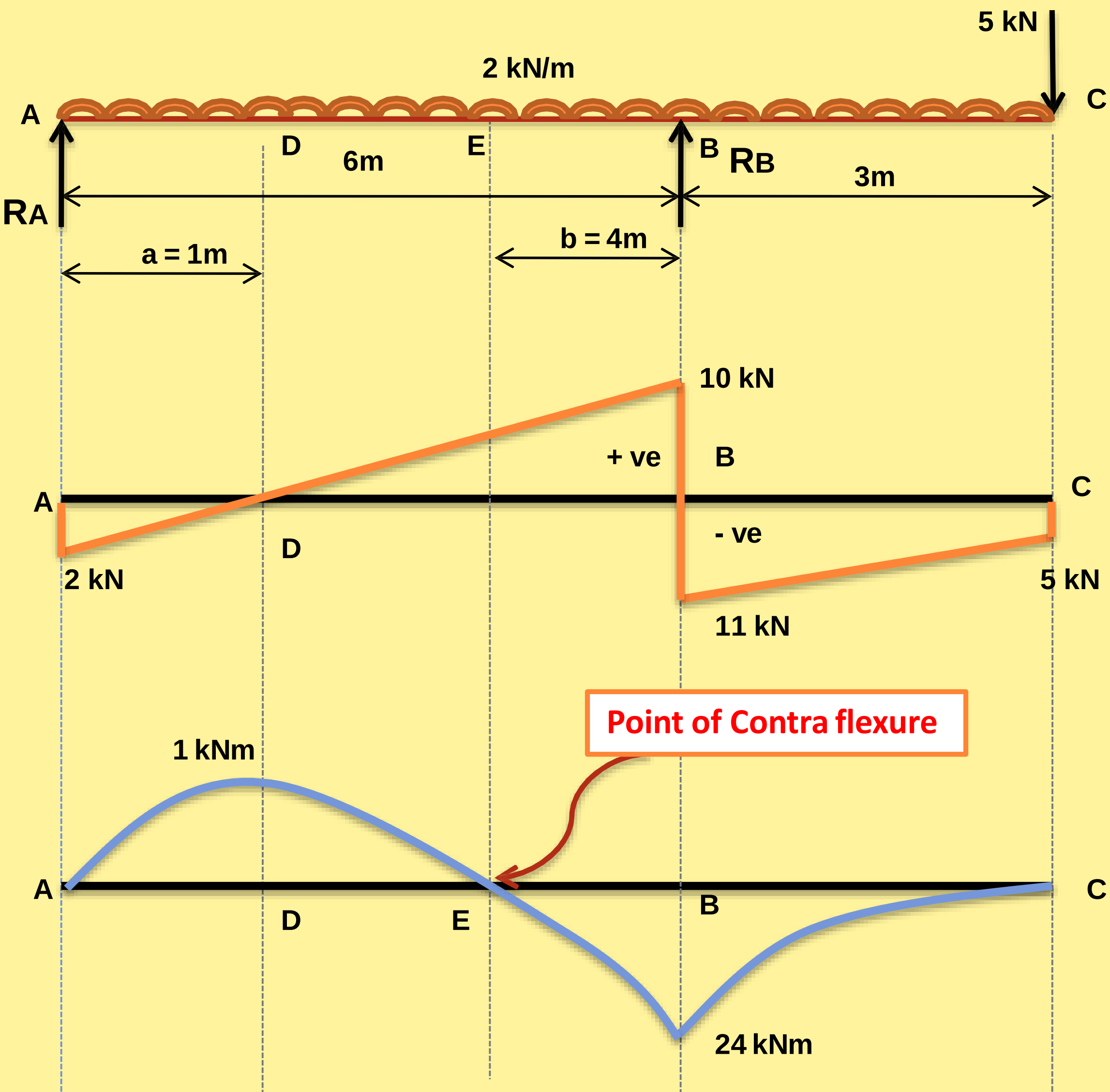
$BM_D = 20 \times 6 - 40 = 80$ kN.m

$BM_E = 20 \times 7 - 40 - 20 \times 1 \times 0.5$

$= 90$ kN.m



OVERHANGING BEAM WITH UDL



BENDING MOMENT DIAGRAM

Calculations

Reactions

M_A,

$R_B \times 6 = 5 \times 9 + 2 \times 9 \times 9/2$

$R_B = 21$ kN

$R_A + R_B = 5 + 2 \times 9 = 23$, $R_A = 2$ kN

Shear Force

S.F at C = 5 kN

S.F at B = 5 + 2 x 3 = -11 kN (just right)

S.F at B = -11 + 21 = 10 kN (just left)

S.F at A = 2 kN (- ve)

S.F between B & A = 0

S.F at D = - 2 + 2 x a

$0 = -2 + 2a$, $a = 1$m

Bending Moment

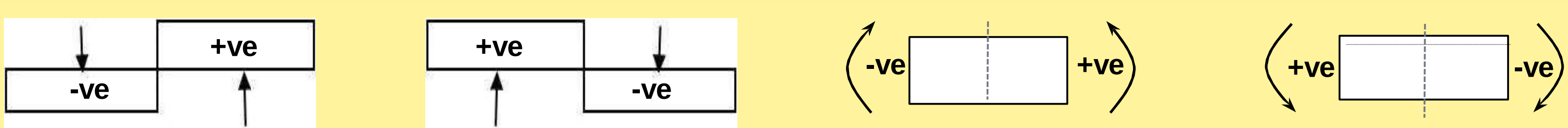
$BM_C = 0$, $BM_A = 0$

$BM_B = -5 \times 3 - 2 \times 3 \times 3/2 = -24$ kN.m

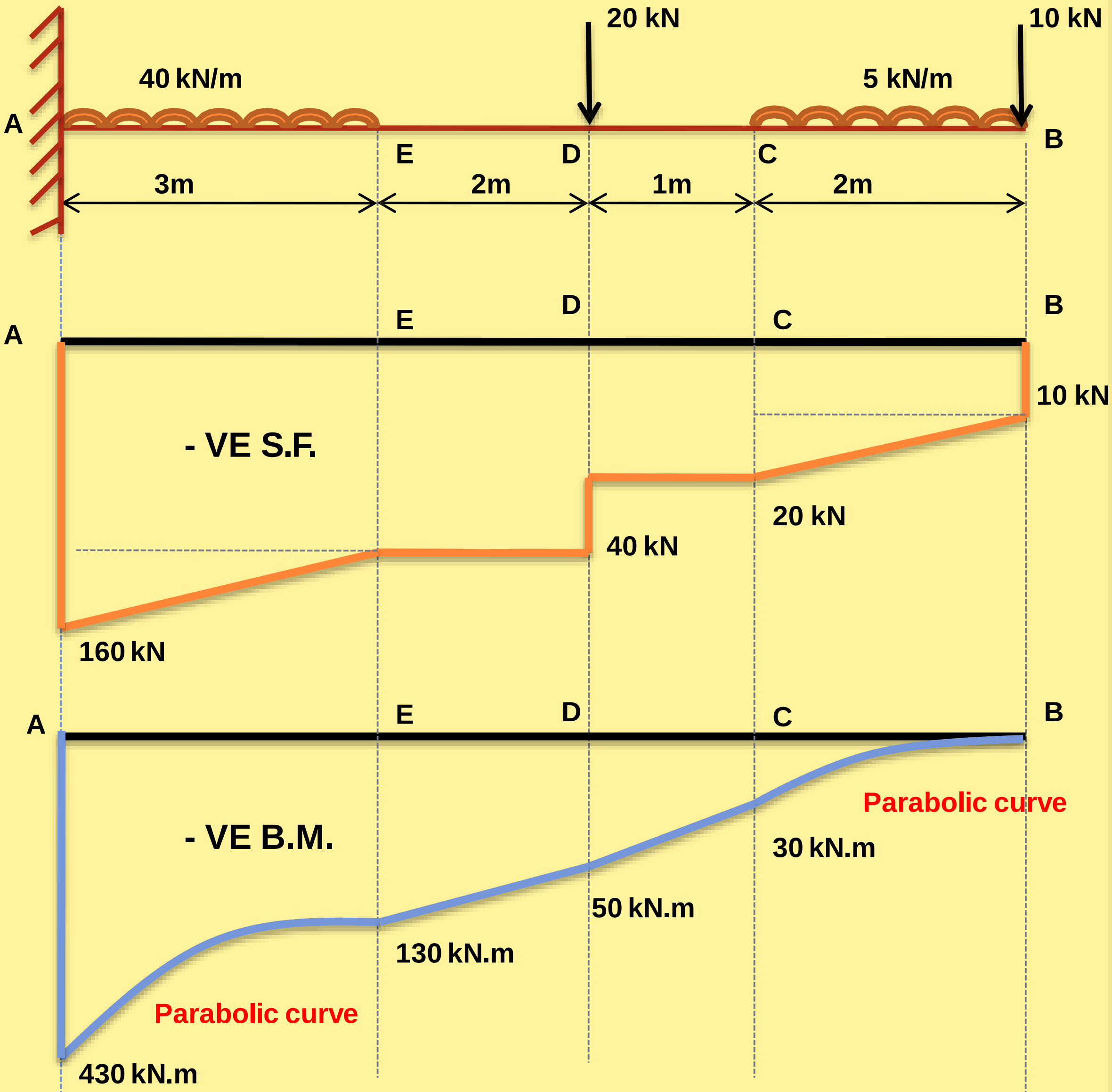
Point of Contra flexure $BM_E = 0$

$-5(b + 3) - 2 \times (b+3)^2 / 2 + 21b = 0$

$b = 4$ m (+ ve value)



CANTILEVER WITH UDL



BENDING MOMENT DIAGRAM

Calculations

Shear Force

S.F at B = 10 kN
S.F at C = 10 + 5 x 2 = -20 kN
S.F at D = 10 + 5 x 2 + 20 = - 40 kN
S.F at A = 10 + 5 x 2 + 20 + 40 x 3
= - 160 kN

Bending Moment

BM_B = 0

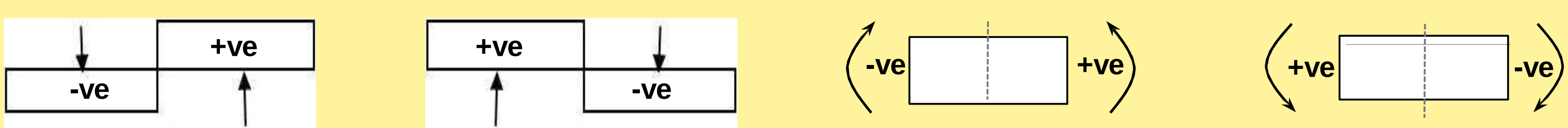
BM_C = 10 x 2 + 5 x 2 x 2/2 = -30 kNm.

BM_D = 10 x 3 + 5 x 2 x (2/2 + 1)
= 50 kN.m

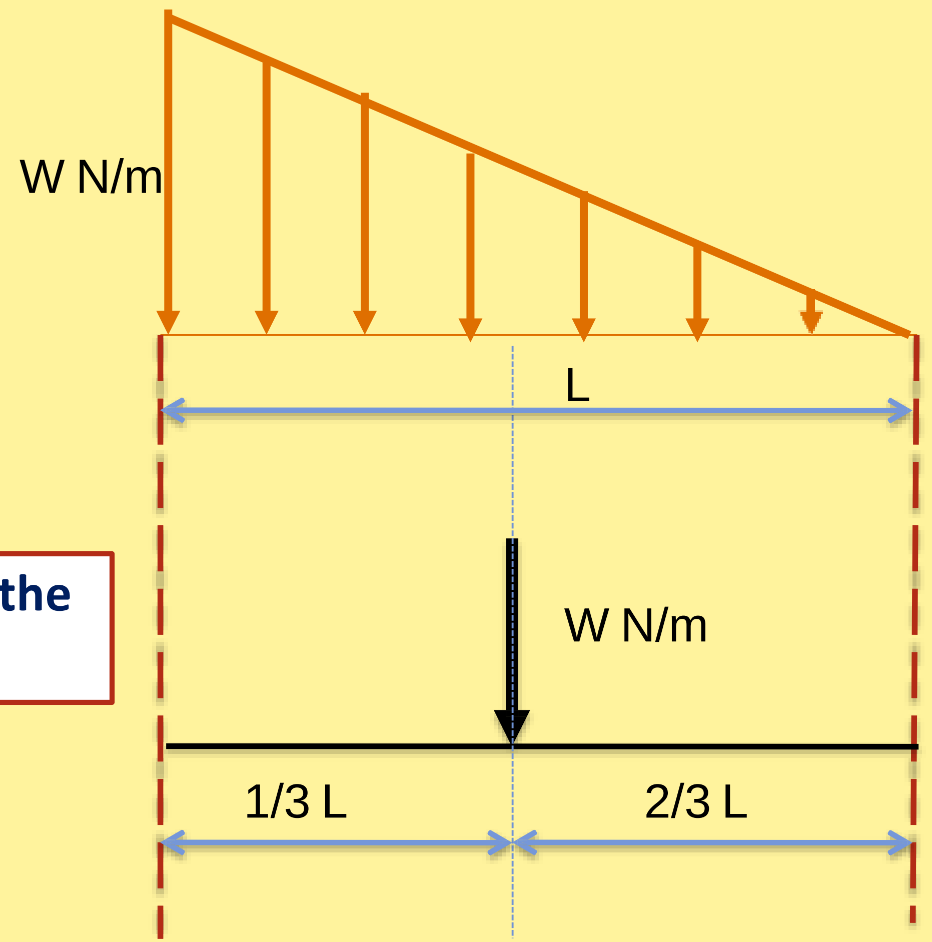
BM_E = 10 x 5 + 5 x 2 x (2/2 + 3) + 20 x 2

= - 130 k Nm

BM_A = 10 x 8 + 5 x 2 x (2/2 + 6) + 20 x 5 + 40 x 3 x 3/2
= - 430 kN.m.

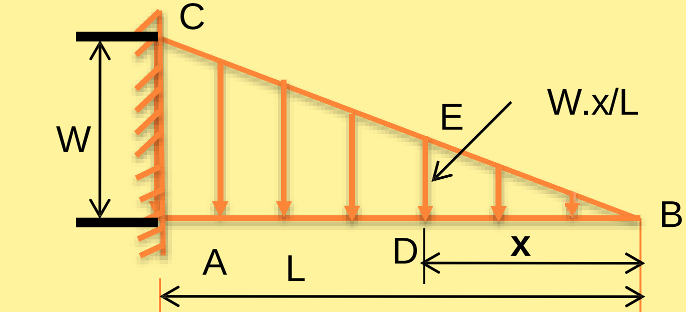


UNIFORMLY VARYING LOAD

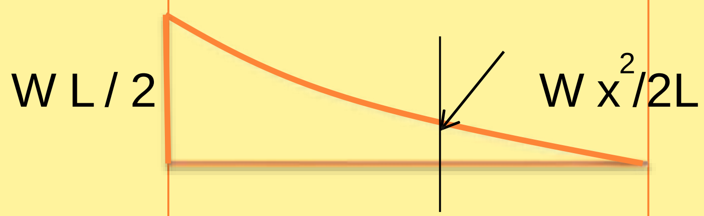


Load Act the centroid of the Triangular Area

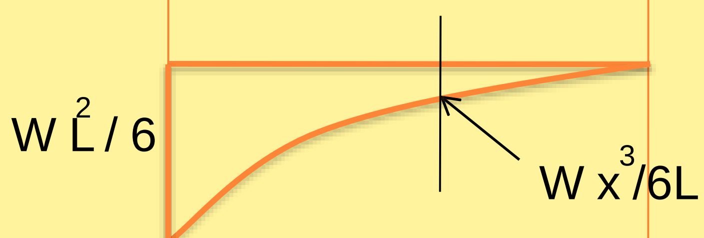
There will be Parabola in both S.F. and B.M



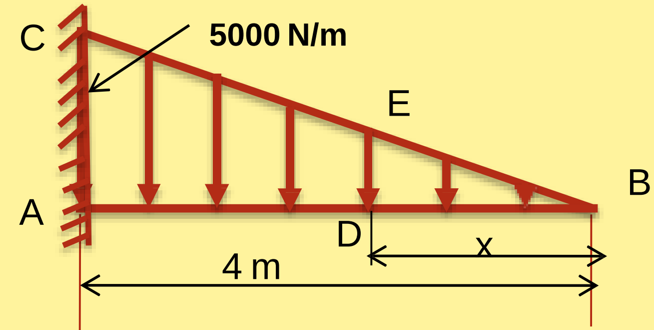
Rate of Loading



S.F. = Triangular Load area = ½ X DE X DB



B.M = Force X Perpendicular
= ½ X DE X DB X DB / 3 from point D to the centroid

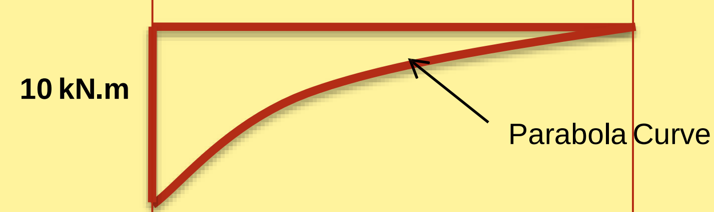


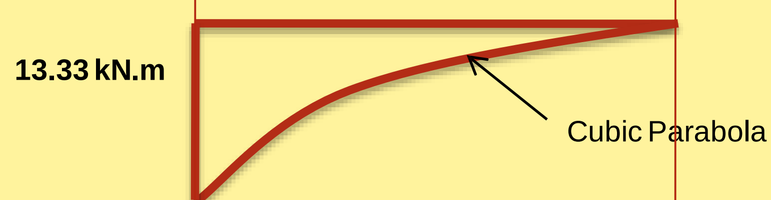
DE / AC = DB / AB , DE = 5000**x**/4 = 1250 **x**
i.e rate loading at any distance x

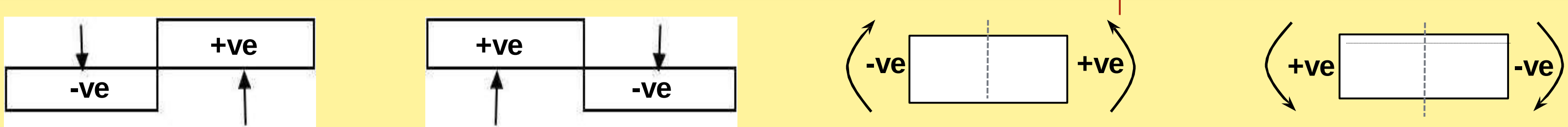
S.F. at D = -1/2 X **x** 1250 **x** = - 625 x^2
S.F. at B = 0 where x= 0
S.F. at A , at x = 4 , -625 X 4^2 = 10 kN

B.M. at x = -1/2 X DB X DE X DB/3

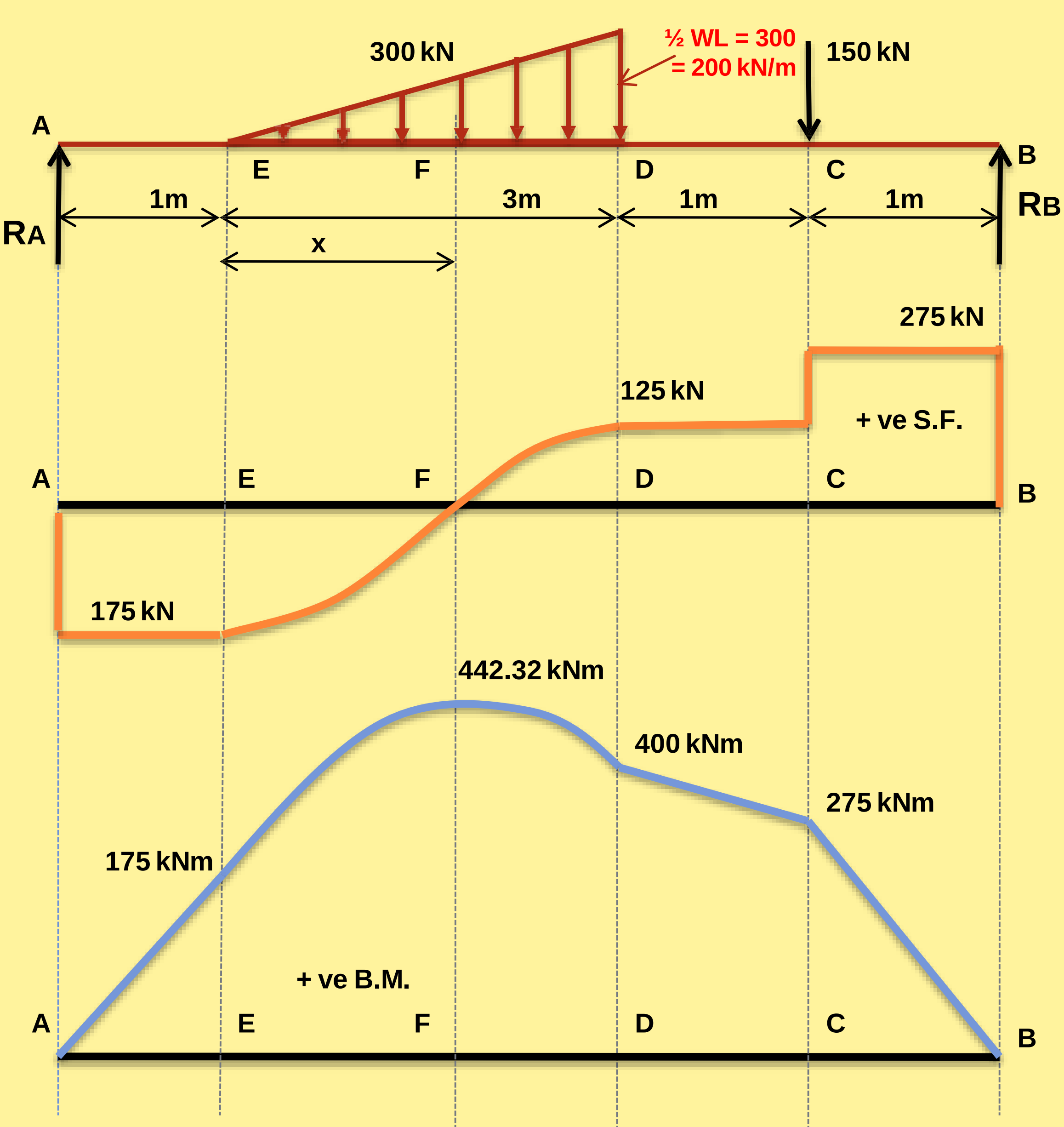
= - 625 x^2X **x**/3, at **x** = 4 ,
B.M. at A = - 13.33 kN.m







Draw S.F. and B.M. diagrams for the loaded Beam



Calculations

Reactions

$R_A + R_B = 150 + 300$

M_A,

$R_B \times 6 = 150 \times 5 + 300\,(2/3 \times 3 + 1)$

$R_B = 275$ kN, $R_A = 175$ kN

Shear Force

S.F at B = 275 kN

S.F at C = 275 – 150 = 125 kN

S.F at D = 125 kN

S.F at E = 275 – 150 – 300 = -175 kN

S.F at A = - 175 kN

Rate of Loading at distance x

w = Wx/L = **w** = 200 **x** / 3

S.F at F = -175 + ½ 200**x** / 3 X **x**

x= 2.29 m

Bending Moment

BM_A = 0, **BM_B** = 0,

BM_C = 275 X 1 = 275 kN.m

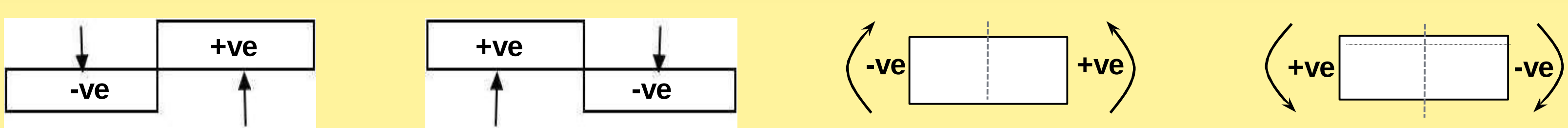
BM_D = 275 X 2 – 150 X 1 = 400 kN.m

BM_E = 175 X 1 = 175 kN.m

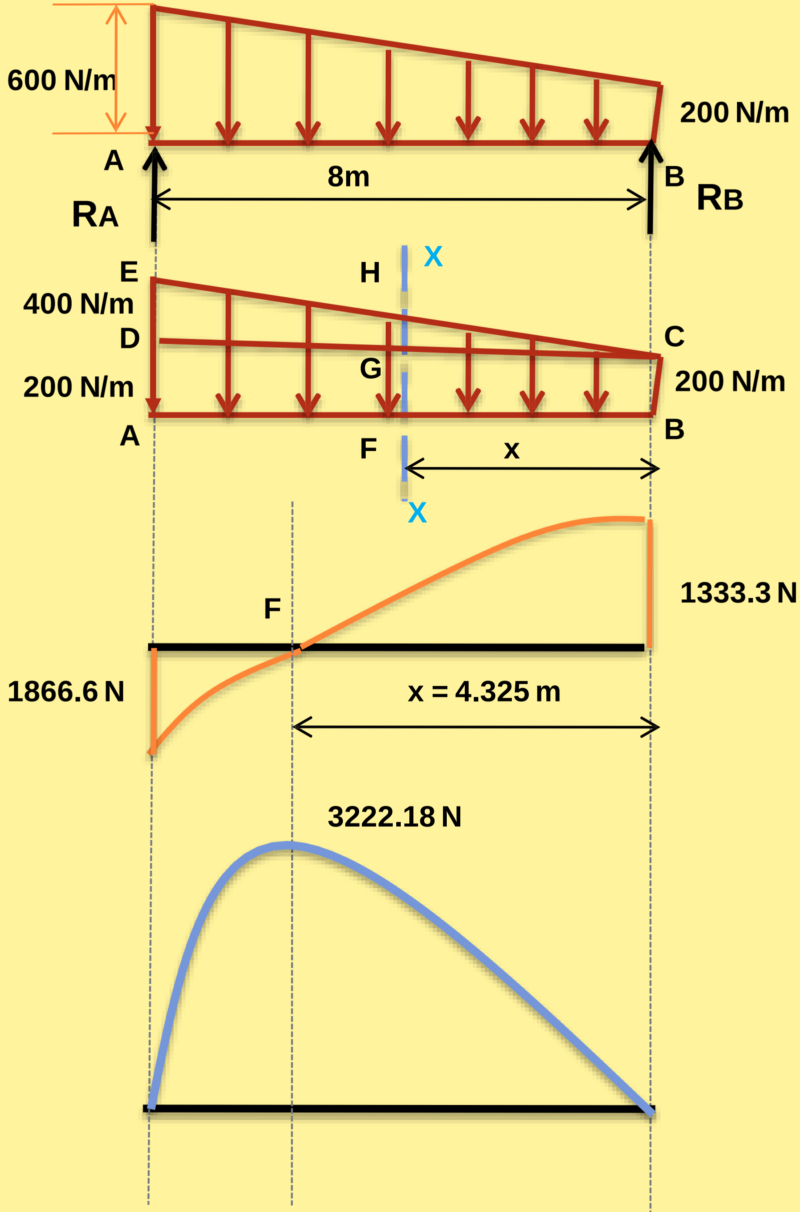
½ WL X L/3

BM_F = 175 X 3.29 – (200 X 2.29/3) (2.29/2 X 2.29/3)

= 442.32 kN.m



Draw S.F. and B.M. diagrams for the loaded Beam



Calculations

Reactions

M_A,

R_B X8= 200 X 8 X 4 + ½ X 400 X 8 X 8/3

R_B = 1333.33 N

R_A + R_B = 200 X 8 + ½ X 400 X 8

R_A = 1866.67 N

Rate of Loading at X-X = GH + GF

Rate of Loading at GH

DE/CD = GH/CG, GH = 400**x** /8 = 50**x**

Rate of Loading at GF = 200

Rate of Loading at X-X = GH + GF = 200 + 50**x**

Shear Force at P = 0

S.F. at F =1333.33 – (load BCGF + Load CGH

= 1333.33 – (200**x** + ½ X 50 **x** X **x)**

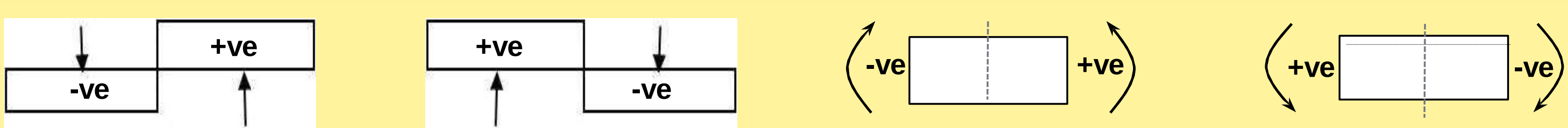
x = 4.326 (quadratic equation +ve value)

Bending Moment

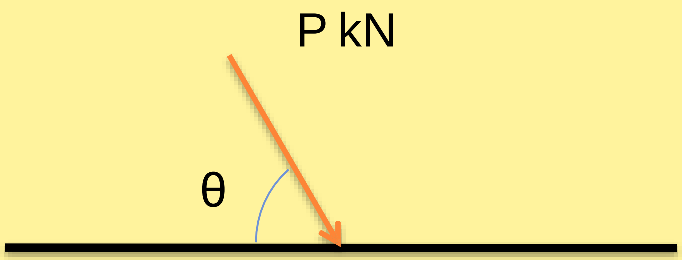
BM$_F$ = 1333.33x – 200x X x/2 – ½ X 50x X x X x/3…….(GH = 50x)

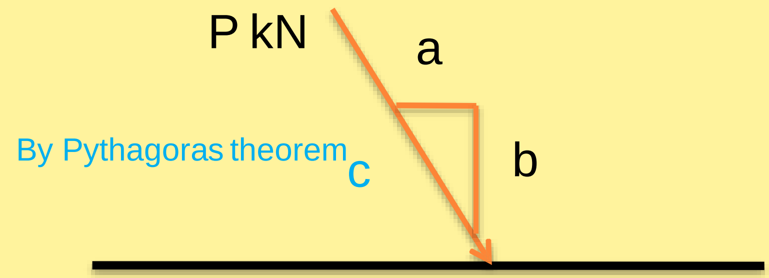
We have x = 4.326

Max. B.M at F = 3436.14 N/m



OBLIQUE OR INCLINED LOAD and Hinges



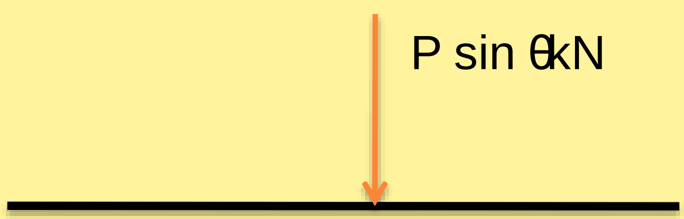


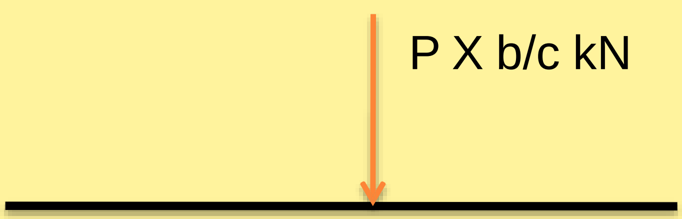
Horizontal Component = $P \cos \theta$ kN

Vertical Component = $P \sin \theta$ kN

Horizontal Component = $P \cos \theta$ kN
= $P \times a/c$ kN

Vertical Component = $P \sin \theta$ kN
= $P \times b/c$ kN





Consider the Vertical Component

For the Hinges consider the Vertical Support Reactions by the help of free body diagram and solve as the regular process generally you will find at the UDL.
At the Hinges you will have Bending Moment is zero.

LOADING EXAMPLES