

# Unit No. 1

## *Properties of Fluids and Dimensional Analysis*

**Fluid** : - Tendency to flow

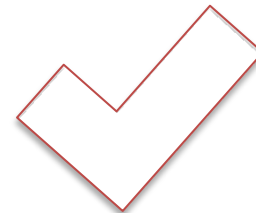
**Ideal Fluids**

(Viscosity, Surface Tension & it is incompressible)

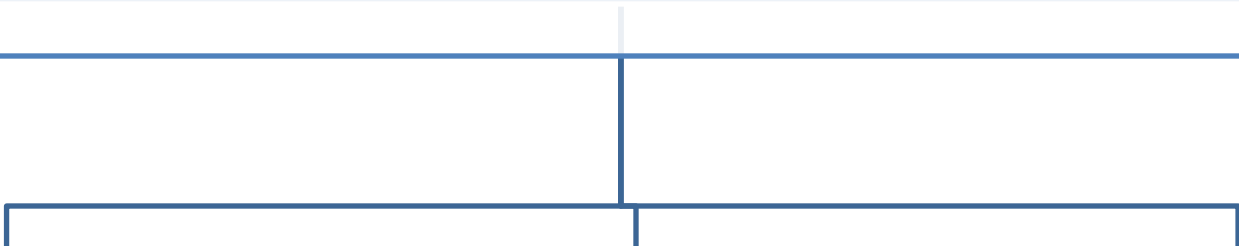


**Real Fluids**

(Viscosity, Surface Tension and possess Compressibility)



**Fluid Mechanics** : - Branch of science that deals with behaviour of fluid at rest as well as in motion.



**Fluid Statics**

(Study of fluids at rest)

**Fluid Kinematics**

(Study of fluids in motion without considering the forces causing the motion)

**Fluid Dynamics**

(Study of fluids in motion with consideration of the forces causing the motion)

# Properties of Fluids

## Mass Density

- Definition  $\rho = \frac{\text{mass of fluid}}{\text{volume of fluid}}$
- Formula :
- Value and units : density of water is 1 gm/cm<sup>3</sup> or 1000 kg/m<sup>3</sup>

## Specific Weight or Weight Density

- Definition  $w = \frac{\text{weight of fluid}}{\text{volume of fluid}}$
- Formula :  
$$= \frac{\text{mass of fluid} * \text{acceleration due to gravity}}{\text{volume of fluid}} \quad w \text{ or } \gamma = \rho * g$$
- Value and units : w for water = 9810 N/m<sup>3</sup>, 9.81 kN/m<sup>3</sup>, 1000 kgf/m<sup>3</sup> or 981 dynes/cm<sup>3</sup>

## Specific Volume

- Definition
- Formula :  
$$\text{specific volume} = \frac{\text{volume of fluid}}{\text{weight of fluid}} = \frac{\text{volume of fluid}}{\text{mass of fluid}}$$
- Units : m<sup>3</sup>/N or m<sup>3</sup>/kgf or cm<sup>3</sup>/dynes  $= \frac{1}{\gamma} = \frac{1}{\rho}$

# Properties of Fluids

## Specific Gravity

- Definition
- Formula :

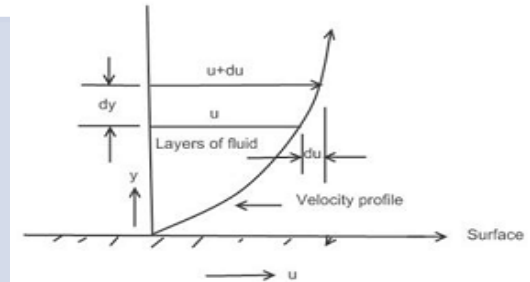
$$\text{specific gravity} = \frac{\text{weight density of fluid}}{\text{weight density of standard fluid}}$$

- Units : Dimensionless / No Unit

## Viscosity

- Definition
- Formula / Derivation :

$$\tau \propto \frac{du}{dy} \quad \tau = \mu \frac{du}{dy}$$



- Units : kg.f-sec/m<sup>2</sup> or dyne-sec/cm<sup>2</sup> or N.s/m<sup>2</sup>
- 1 N.s/m<sup>2</sup> = 10 poise
- Dynamic Viscosity :
- Kinematic Viscosity :
- Newtons Law of Viscosity

$$\mu = \frac{\tau}{\frac{du}{dy}} \quad \nu = \frac{\mu}{\rho}$$

Fluid

obey  
--->

Newton's law  
of viscosity

refer  
--->

Newtonian fluids

Fluid

Do not obey  
--->

Newton's law  
of viscosity

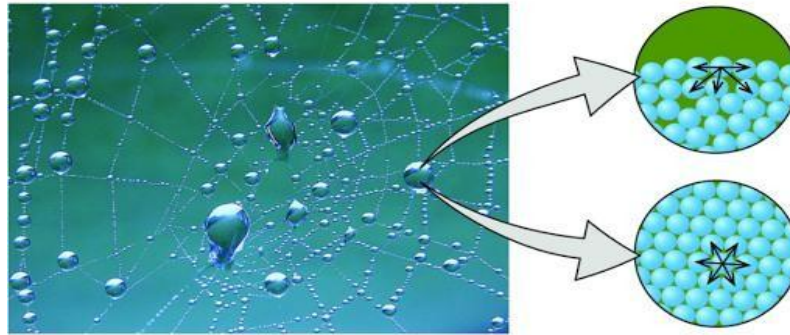
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Non-Newtonian  
fluids

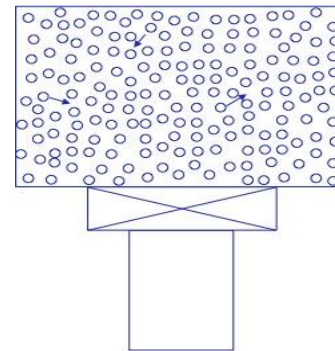
# Properties of Fluids

- **Variation of Viscosity with Temperature** : Viscosity of liquids decreases with increase in temperature while viscosity of gases increases with increase in temperature.

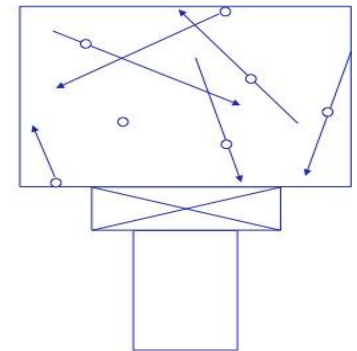
- **Cohesive Forces**



- **Molecular Momentum Transfer**



**Viscous Flow**  
(momentum transfer  
between molecules)



**Molecular Flow**  
(molecules move  
independently)

# Properties of Fluids

- Relation between viscosity and temperature :

1. For liquids :

$$\mu = \mu_0 \left( \frac{1}{1 + \alpha t + \beta t^2} \right)$$

$\mu$  = viscosity of liquid at  $t$  °C in poise.      For water :  $\mu_0 = 1.79 \times 10^{-3}$  poise

$\mu_0$  = viscosity of liquid at 0 °C in poise.       $\alpha = 0.03368$

$\alpha, \beta$  = constants for liquids/gases.       $\beta = 0.000221$

2. For Gases :

$$\mu = \mu_0 + \alpha t - \beta t^2$$

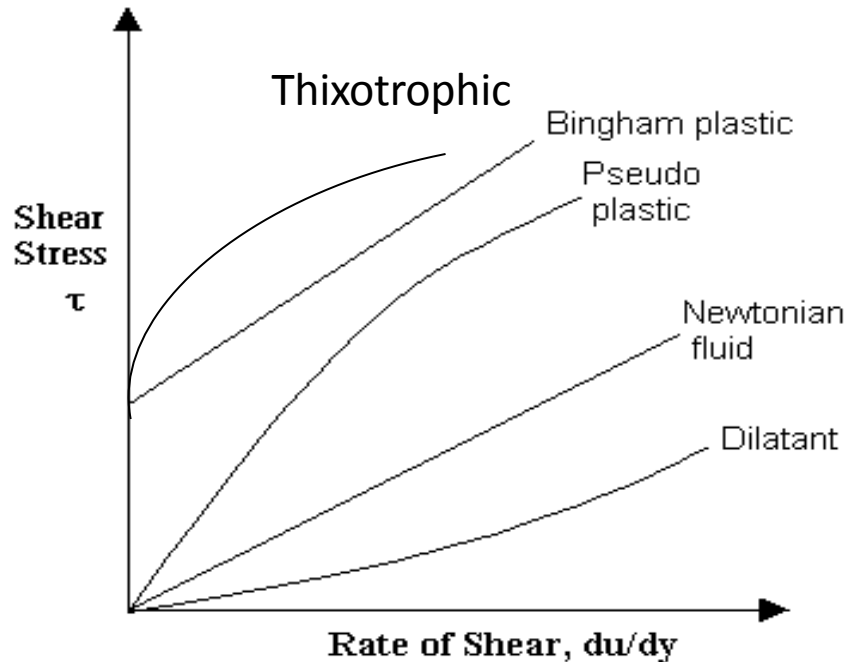
For air :  $\mu_0 = 0.000017$  poise

$\alpha = 5.6 \times 10^{-8}$

$\beta = 0.1189 \times 10^{-9}$

# Properties of Fluids

- Types of Fluids :



**Bingham plastic** : resist a small shear stress but flow easily under large shear stresses, e.g. sewage sludge, toothpaste, and jellies.

**Pseudo plastic** : most non-Newtonian fluids fall under this group. Viscosity decreases with increasing velocity gradient, e.g. colloidal substances like clay, milk, and cement.

**Dilatants** : viscosity decreases with increasing velocity gradient, e.g. quicksand.

**Thixotropic** : non-linear relationship between the shear stress and the rate of angular deformation, beyond an initial yield stress



# Properties of Fluids

## Compressibility

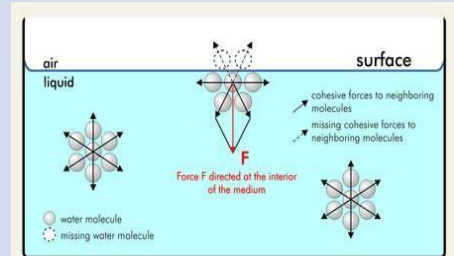
- Definition
- Formula :  $1 / \text{Bulk Modulus (K)}$

## Bulk Modulus

- Definition
- Formula :  $\text{Bulk modulus} = \frac{\text{compressive stress}}{\text{volumetric strain}} = \frac{-dp}{\frac{dv}{v}}$
- Effect of T & P :  $dp \uparrow$   $K \uparrow$  and  $T \uparrow$   $K \downarrow$  (liquids)
- $T \uparrow$   $P \uparrow$   $K \uparrow$  (gases)
- Isothermal Process :  $p = K$
- Adiabatic process :  $K = pk$

## Surface Tension

- Definition
- Formula : 1. Liquid Droplet :  $p = 4\sigma/d$
- 2. Hollow Bubble :  $p = 8\sigma/d$
- 3. Liquid Jet :  $p = 2\sigma/d$
- Units :  $\text{N/cm}^2$



# Properties of Fluids

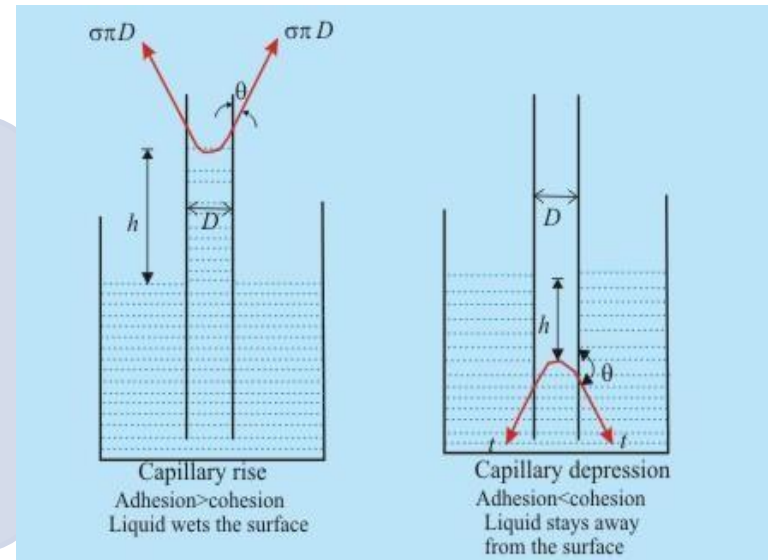
## Capillarity

- Definition
- Expression for capillary Rise:

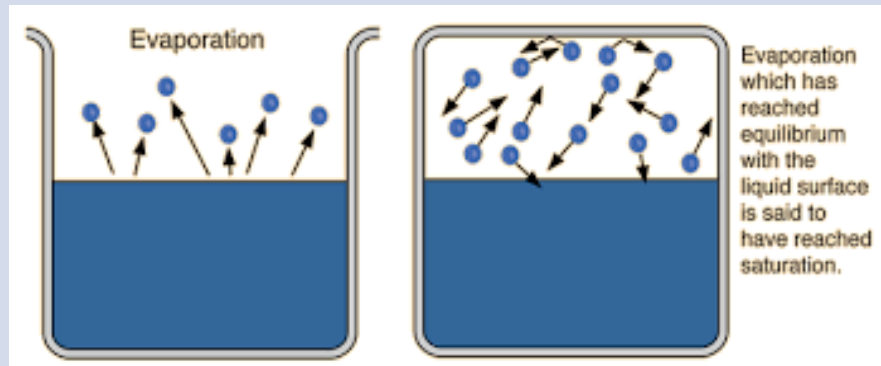
$$h = \frac{4\sigma}{\rho g d}$$

- Expression for capillary Fall :

$$h = \frac{4\sigma \cos \theta}{\rho g d}$$



## Vapour Pressure & Cavitation



# Dimensional Analysis

- Method of Dimensions.
- **Mathematical Technique** used in research work for design and conducting model tests.
- Deals with the **dimensions of physical quantities** involved in the phenomenon.
- All physical quantities are measured by **comparison** with respect to an arbitrarily fixed value.
- Length **L**, Mass **M** and Time **T** are three fixed dimensions which are of importance in fluid mechanics.
- These fixed dimensions are called as **Fundamental Dimensions** or **Fundamental Quantities**.
- The quantities those are derived from these fundamental quantities are known as **Secondary or Derived Quantities**. They possess more than one fundamental dimensions.

# Dimensional Analysis

S.No.	Physical Quantity	Relation with other physical quantities	Dimensional formula	SI-Unit
1	Area	Length x breadth	$[L] \times [L] = [M^0 L^2 T^0]$	$m^2$
2	Volume	Length x breadth x height	$[L] \times [L] \times [L] = [M^0 L^3 T^0]$	$m^3$
3	Density	$\frac{\text{mass}}{\text{volume}}$	$\frac{[M]}{[L^3]} = [M L^{-3} T^0]$	$Kg\ m^{-3}$
4	Speed or velocity	$\frac{\text{distance}}{\text{time}}$	$\frac{[L]}{[T]} = [M^0 L T^{-1}]$	$m\ s^{-1}$
5	Acceleration	$\frac{\text{velocity}}{\text{time}}$	$\frac{[L T^{-1}]}{[T]} = [M^0 L T^{-2}]$	$m\ s^{-2}$
6	Momentum	Mass x velocity	$[M] \times [L T^{-1}] = [M L T^{-1}]$	$Kg\ m\ s^{-1}$
7	Force	Mass x acceleration	$[M] \times [L T^{-2}] = [M L T^{-2}]$	N (Newton)
8	Pressure	$\frac{\text{force}}{\text{area}}$	$\frac{[M L T^{-2}]}{[L^2]} = [M L^{-1} T^{-2}]$	$N\ m^{-2}$ or Pa (Pascal)
9	Work	Force x distance	$[M L T^{-2}] \times [L] = [M L^2 T^{-2}]$	J (Joule)
10	Energy	Work	$[M L^2 T^{-2}]$	J
11	Power	$\frac{\text{work}}{\text{time}}$	$\frac{[M L^2 T^{-2}]}{[T]} = [M L^2 T^{-3}]$	W (Watt)
12	Gravitational constant (G)	$\frac{\text{force} \times \text{distance}^2}{\text{mass}^2}$	$[M^{-1} L^3 T^{-2}]$	$N\ m^2\ kg^{-2}$
13	Impulse	Force x time	$[M L T^{-2}] \times [T] = [M L T^{-1}]$	N s
14	Surface tension	$\frac{\text{force}}{\text{length}}$	$\frac{[M L T^{-2}]}{[L]} = [M L^0 T^{-2}]$	$N\ m^{-1}$
15	Coefficient of viscosity	$\frac{\text{force}}{\text{area} \times \text{velocity gradient}}$	$[M L^{-1} T^{-1}]$	daP (decapoise)
16	Angle	$\frac{\text{arc}}{\text{radius}}$	Dimensionless	rad
17	Moment of inertia	Mass x distance <sup>2</sup>	$[M L^2 T^0]$	$Kg\ m^2$
18	Angular momentum	Moment of inertia x angular velocity	$[M L^2] \times [T^{-1}] = [M L^2 T^{-1}]$	$Kg\ m^2\ s^{-1}$
19	Torque or couple	Force x perpendicular distance	$[M L T^{-2}] \times [L] = [M L^2 T^{-2}]$	N m
20	Frequency	$\frac{1}{\text{second}}$	$[T^{-1}]$	Hz

# Dimensional Analysis

- **Dimensional Homogeneity** : It means the dimensions of each term in an equation on both sides are equal.
- Dimensions of L.H.S = Dimensions of R.H.S

*Methods of Dimensional Analysis*

*Rayleigh's Method*

*Buckingham's  $\pi$ -Theorem*

# Rayleigh's Method

- To define **relationship** among the variables.
- This method is used for determining the **expression** for a variable which depends upon maximum **three** or **four** variables only.

Methodology:

Let  $X$  is a function of  $X_1, X_2, X_3$  and mathematically it can be written as  
$$X = f(X_1, X_2, X_3)$$

This can be also written as

$$X = K (X_1^a, X_2^b, X_3^c)$$
 where  $K$  is constant and  $a, b$  and  $c$  are arbitrarily powers

The values of  $a, b$  and  $c$  are obtained by comparing the powers of the fundamental dimension on both sides.

- Thus the expression is obtained for dependent variable.

# Rayleigh's Method

**Problem :** Find the equation for the power developed by a pump if it depends on head  $H$  discharge  $Q$  and specific weight  $\gamma$  of the fluid.

**Solution:**

$$P = f(H, Q, \gamma)$$

$$P = K \cdot H^a \cdot Q^b \cdot \gamma^c$$

$$[P] = [H]^a \cdot [Q]^b \cdot [\gamma]^c$$

$$[L^2MT^{-3}] = [LM^0T^0]^a \cdot [L^3M^0T^{-1}]^b \cdot [L^{-2}MT^{-2}]^c$$

Power	$= L^2MT^{-3}$
Head	$= LM^0T^0$
Discharge	$= L^3M^0T^{-1}$
Specific Weight	$= L^{-2}MT^{-2}$

Equating the powers of M, L and T on both sides,

Power of M,  $1 = c$

Power of T,  $-3 = -b - 2$  or  $b = -2 + 3$  or  $b = 1$

Power of L,  $2 = a + 3b - 2c$  or  $2 = a + 3 - 2$  or  $a = 1$

Substituting the values of a, b and c

$$P = K \cdot H^1 \cdot Q^1 \cdot \gamma^1$$

$$P = K \cdot H \cdot Q \cdot \gamma \quad \text{When, } K = 1 \quad P = H \cdot Q \cdot \gamma$$



# Rayleigh's Method

**Problem 3:** Find an expression for drag force  $R$  on a smooth sphere of diameter  $D$  moving with uniform velocity  $V$  in a fluid of density  $\rho$  and dynamic viscosity  $\mu$ .

**Solution:**

$$R = f(D, V, \rho, \mu)$$

$$R = K \cdot D^a \cdot V^b \cdot \rho^c \cdot \mu^d$$

$$[R] = [D]^a \cdot [V]^b \cdot [\rho]^c \cdot [\mu]^d$$

$$[LMT^{-2}] = [LM^0T^0]^a \cdot [LM^0T^{-1}]^b \cdot [L^{-3}MT^0]^c \cdot [L^{-1}MT^{-1}]^d$$

$$\text{Force} = LMT^{-2}$$

$$\text{Diameter} = LM^0T^0$$

$$\text{Velocity} = LM^0T^{-1}$$

$$\text{Mass density} = L^3MT^0$$

Equating the powers of M, L and T on both sides,

$$\text{Power of M,} \quad 1 = c + d \quad \text{or} \quad \mathbf{c = 1 - d}$$

$$\text{Power of T,} \quad -2 = -b - d \quad \text{or} \quad \mathbf{b = 2 - d}$$

$$\text{Power of L,} \quad 1 = a + b - 3c - d \quad \text{or} \quad 1 = a + 2 - d - 3(1 - d) - d$$

$$1 = a + 2 - d - 3 + 3d - d \quad \text{or} \quad \mathbf{a = 2 - d}$$

Substituting the values of a, b, and c

$$R = K \cdot D^{2-d} \cdot V^{2-d} \cdot \rho^{1-d} \cdot \mu^d = K \frac{D^2}{D^d} \cdot \frac{V^2}{V^d} \cdot \frac{\rho}{\rho^d} \cdot \mu^d$$

$$= K \cdot \rho V^2 D^2 \left[ \frac{\mu}{\rho V D} \right]^d = \rho V^2 D^2 \phi \left[ \frac{\mu}{\rho V D} \right]$$



# Buckingham's $\pi$ -Theorem

This method of analysis is used when number of variables are more.

## Theorem:

If there are  $n$  variables in a physical phenomenon and those  $n$  variables contain  $m$  dimensions, then variables can be arranged into  $(n-m)$  dimensionless groups called  $\pi$  terms.

## Explanation:

If  $f(X_1, X_2, X_3, \dots, X_n) = 0$  and variables can be expressed using  $m$  dimensions then

$f(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = 0$  where,  $\pi_1, \pi_2, \pi_3, \dots$  are dimensionless groups.

Each  $\pi$  term contains  $(m + 1)$  variables out of which  $m$  are of repeating type and one is of non-repeating type.

Each  $\pi$  term being dimensionless, the dimensional homogeneity can be used to get each  $\pi$  term.

$\pi$  denotes a non-dimensional parameter

# Buckingham's $\pi$ -Theorem

## Method of Selecting Repeating Variables :

1. The dependent variable should not be selected as repeating variable.
2. Repeating variables selected should not form a dimensionless group.
3. Repeating variables together must have same number of fundamental dimensions.
4. No two repeating variables should have same dimensions.
5. Repeating variables should be selected from each of the following properties
  - i. Geometric Property : Length, height, width, area.
  - ii. Flow Property : Velocity, Acceleration, Discharge.
  - iii. Fluid Property : Mass density, Viscosity, Surface tension.

# Buckingham's $\pi$ -Theorem

**Problem 12.11** The pressure difference  $\Delta p$  in a pipe of diameter  $D$  and length  $l$  due to viscous flow depends on the velocity  $V$ , viscosity  $\mu$  and density  $\rho$ . Using Buckingham's  $\pi$ -theorem, obtain an expression for  $\Delta p$ .

**Solution.**

$\Delta p$  is a function of  $D, l, V, \mu, \rho$  or  $\Delta p = f(D, l, V, \mu, \rho)$

or  $f_1(\Delta p, D, l, V, \mu, \rho) = 0$  ... (i)

Total number of variables,  $n = 6$

Number of fundamental dimension,  $m = 3$

Number of  $\pi$ -terms  $= n - 3 = 6 - 3 = 3$

Hence equation (i) is written as  $f_1(\pi_1, \pi_2, \pi_3) = 0$  ... (ii)

Each  $\pi$ -term contains  $m + 1$  variables, i.e.,  $3 + 1 = 4$  variable. Out of four variables, three are repeating variables.

Choosing  $D, V, \mu$  as repeating variables, we have  $\pi$ -terms as

$$\pi_1 = D^{a_1} \cdot V^{b_1} \cdot \mu^{c_1} \cdot \Delta p$$

$$\pi_2 = D^{a_2} \cdot V^{b_2} \cdot \mu^{c_2} \cdot l$$

$$\pi_3 = D^{a_3} \cdot V^{b_3} \cdot \mu^{c_3} \cdot \rho$$

# Buckingham's $\pi$ -Theorem

**First  $\pi$ -term**  $\pi_1 = D^{a_1} \cdot V^{b_1} \cdot \mu^{c_1} \cdot \Delta p$

Substituting the dimensions on both sides,

$$M^0 L^0 T^0 = L^{a_1} \cdot (LT^{-1})^{b_1} \cdot (ML^{-1}T^{-1})^{c_1} \cdot ML^{-1}T^{-2}$$

Equating the powers of  $M, L, T$  on both sides,

Power of  $M$ ,  $0 = c_1 + 1, \quad \therefore c_1 = -1$

Power of  $L$ ,  $0 = a_1 + b_1 - c_1 - 1, \quad \therefore a_1 = -b_1 + c_1 + 1 = 1 - 1 + 1 = 1$

Power of  $T$ ,  $0 = -b_1 - c_1 - 2, \quad \therefore b_1 = -c_1 - 2 = 1 - 2 = -1$

Substituting the values of  $a_1, b_1$  and  $c_1$  in  $\pi_1$ ,

$$\pi_1 = D^1 \cdot V^{-1} \cdot \mu^{-1} \cdot \Delta p = \frac{D\Delta p}{\mu V}$$

**Second  $\pi$ -term**  $\pi_2 = D^{a_2} \cdot V^{b_2} \cdot \mu^{c_2} \cdot l$

Substituting the dimensions on both sides,

$$M^0 L^0 T^0 = L^{a_2} \cdot (LT^{-1})^{b_2} \cdot (ML^{-1}T^{-1})^{c_2} \cdot L$$

Equating the powers of  $M, L, T$  on both sides

Power of  $M$ ,  $0 = c_2, \quad \therefore c_2 = 0$

Power of  $L$ ,  $0 = a_2 + b_2 - c_2 + 1, \quad \therefore a_2 = -b_2 + c_2 - 1 = -1$

Power of  $T$ ,  $0 = -b_2 - c_2, \quad \therefore b_2 = -c_2 = 0$

Substituting the values of  $a_2, b_2$  and  $c_2$  in  $\pi_2$ ,

$$\pi_2 = D^{-1} \cdot V^0 \cdot \mu^0 \cdot l = \frac{l}{D}$$

# Buckingham's $\pi$ -Theorem

**Third  $\pi$ -term**  $\pi_3 = D^{a_3} \cdot V^{b_3} \cdot \mu^{c_3} \cdot \rho$

Substituting the dimension on both sides,

$$M^0 L^0 T^0 = L^{a_3} \cdot (LT^{-1})^{b_3} \cdot (ML^{-1}T^{-1})^{c_3} \cdot ML^{-3}$$

Equating the powers of  $M, L, T$  on both sides

Power of  $M$ ,  $0 = c_3 + 1, \quad \therefore c_3 = -1$

Power of  $L$ ,  $0 = a_3 + b_3 - c_3 - 3, \quad \therefore a_3 = -b_3 + c_3 + 3 = -1 - 1 + 3 = 1$

Power of  $T$ ,  $0 = -b_3 - c_3, \quad \therefore b_3 = -c_3 = -(-1) = 1$

Substituting the values of  $a_3, b_3$  and  $c_3$  in  $\pi_3$ ,

$$\pi_3 = D^1 \cdot V^1 \cdot \mu^{-1} \cdot \rho = \frac{\rho DV}{\mu}$$

Substituting the values of  $\pi_1, \pi_2$  and  $\pi_3$  in equation (ii),

$$f_1 \left( \frac{D\Delta p}{\mu V}, \frac{l}{D}, \frac{\rho DV}{\mu} \right) = 0 \quad \text{or} \quad \frac{D\Delta p}{\mu V} = \phi \left[ \frac{l}{D}, \frac{\rho DV}{\mu} \right] \quad \text{or} \quad \Delta p = \frac{\mu V}{D} \phi \left[ \frac{l}{D}, \frac{\rho DV}{\mu} \right]$$

Experiments show that the pressure difference  $\Delta p$  is a linear function  $\frac{l}{D}$ . Hence  $\frac{l}{D}$  can be taken out of the functional as

$$\Delta p = \frac{\mu V}{D} \times \frac{l}{D} \phi \left[ \frac{\rho DV}{\mu} \right]. \text{ Ans.}$$

# Model Analysis

- For predicting the **performance** of the hydraulic structures (such as dams, spillways etc.) or hydraulic machines (such as turbines, pumps etc.) before actually constructing or manufacturing, models of the structures or machines are made and **tests** are conducted on them to obtain the desired information.
- **Model** is a small replica of the actual structure or machine.
- The actual structure or machine is called as **Prototype**.
- Models can be smaller or larger than the Prototype.
- **Model Analysis** is actually an experimental method of finding solutions of complex flow problems.

# Model Analysis

- Advantages of Dimensional and Modal Analysis :
  1. Performance of the hydraulic structure or hydraulic machine can be easily predicted, in advance from its model.
  2. With the help of D.A, a relationship between the variables influencing a flow problem in terms of dimensionless parameters is obtained. This relationship helps in conducting tests on the models.
  3. Merits of alternative designs can be predicted with the help of model testing. The most and safe design is finally adopted.
  4. The tests performed on the models can be utilized for obtaining useful information about the performance of the prototypes.
- This can be obtained only if similarity exists between the model and prototype.

# Similitude or Similarities

- Similitude is defined as the **similarity** between the **model** and **prototype** in every aspect, which means that the model and prototype have **similar properties**.
  
- Types of Similarities:
  1. **Geometric Similarity** : Length, Breadth, Depth, Diameter, Area, Volume etc.
  2. **Kinematic Similarity** : Velocity, Acceleration etc.,
  3. **Dynamic Similarity** : Time, Discharge, Force, Pressure Intensity, Torque, Power



# Similitude or Similarities

1. **Geometric Similarity** : The geometric similarity is said to exist between the model and prototype if the **ratio** of all corresponding **linear dimensions** in the model and prototype are equal.

$$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} = L_r$$

$$\frac{A_p}{A_m} = L_r^2$$

$$\frac{V_p}{V_m} = L_r^3$$

where  $L_r$  is Scale Ratio

# Similitude or Similarities

2. **Kinematic Similarity** : The kinematic similarity is said to be exist between model and prototype if the **ratios** of **velocity** and **acceleration** at corresponding points in the model and at the corresponding points in the prototype are the same.

$$\frac{V_p}{V_m} = V_r$$

where  $V_r$  is Velocity Ratio

$$\frac{a_p}{a_m} = a_r$$

where  $a_r$  is Acceleration Ratio

Also the directions of the velocities in the model and prototype should be same

# Similitude or Similarities

3. **Dynamic Similarity** : The dynamic similarity is said to be exist between model & prototype if the **ratios** of corresponding **forces** acting at the corresponding points are Equal.

$$\frac{F_p}{F_m} = F_r$$

where  $F_r$  is Force Ratio

- It means for dynamic similarity between the model and prototype, the **dimensionless numbers** should be same for model and prototype.

# Types of Forces Acting on Moving Fluid

1. **Inertia Force ( $F_i$ )** : It is the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration.
  - It always exists in the fluid flow problems.
2. **Viscous Force ( $F_v$ )** : It is equal to the product of shear stress due to viscosity and surface area of the flow.
3. **Gravity Force ( $F_g$ )** : It is equal to the product of mass and acceleration due to gravity of the flowing fluid.
4. **Pressure Force ( $F_p$ )** : It is equal to the product of pressure intensity and cross sectional area of flowing fluid.
5. **Surface Tension Force ( $F_s$ )** : It is equal to the product of surface tension and length of surface of the flowing.
6. **Elastic Force ( $F_e$ )** : It is equal to the product of elastic stress and area of the flowing fluid.

# Dimensionless Numbers

- Dimensionless numbers are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force.

1. Reynold's number,  $R_e =$

$$\frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{\rho V L}{\mu} \text{ or } \frac{\rho V D}{\mu}$$

2. Froude's number,  $F_e =$

$$\sqrt{\frac{\text{Inertia Force}}{\text{Gravity Force}}} = \frac{V}{\sqrt{Lg}}$$

3. Euler's number,  $E_u =$

$$\sqrt{\frac{\text{Inertia Force}}{\text{Pressure Force}}} = \frac{V}{\sqrt{p/\rho}}$$

4. Weber's number,  $W_e =$

$$\sqrt{\frac{\text{Inertia Force}}{\text{Surface Tension Force}}} = \frac{V}{\sqrt{\sigma/\rho L}}$$

5. Mach's number  $M =$

$$\sqrt{\frac{\text{Inertia Force}}{\text{Elastic Force}}} = \frac{V}{C}$$

# Model Laws

- The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity.

1. **Reynold's Model Law** : Model law in which models are based on Reynold's number.

Models based on Reynolds's Number includes:

- a) Pipe Flow.
- b) Resistance experienced by Sub-marines, airplanes, fully immersed bodies.

2. **Froude Model Law** : Model law in which models are based on Froude's number.

Froude Model Law is applied in the following fluid flow problems:

- a) Free Surface Flows such as Flow over spillways, Weirs, Sluices, Channels etc.
- b) Flow of jet from an orifice or nozzle.
- c) Where waves are likely to formed on surface.

# Reynold's Model Law

- If the **viscous forces** are predominant, the models are designed for dynamic similarity based on Reynold's number.

$$[Re]_m = [Re]_p$$

$$t_r = \text{Time Scale Ratio} = \frac{L_r}{V_r}$$

Velocity,  $V = \text{Length/Time} \rightarrow T = L/V$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$a_r = \text{Acceleration Scale Ratio} = \frac{V_r}{t_r}$$

Acceleration,  $a = \text{Velocity/Time} \rightarrow L = V/T$

# Froude Model Law

- If the **gravity force** is predominant, the models are designed for dynamic similarity based on Froude number.

$$[F_e]_m = [F_e]_p \Rightarrow \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}} \Rightarrow V_r = \text{Velocity Scale Ratio} = \sqrt{L_r}$$

$$T_r = \text{Scale Ratio for Time} = \sqrt{L_r}$$

$$F_r = \text{Scale Ratio for Force} = L_r^3$$

$$T_r = \text{Scale Ratio for Acceleration} = 1$$

$$F_r = \text{Scale Ratio for Pressure Intensity} = L_r$$

$$Q_r = \text{Scale Ratio for Discharge} = L_r^{2.5}$$

$$P_r = \text{Scale Ratio for Power} = L_r^{3.5}$$



# Summary

- 10 properties of fluids with numerical.
- Effect of temperature and pressure on all the properties of fluids.
- Rheological diagram and types of fluids.
- Fundamental and secondary quantities.
- Dimensional Homogeneity.
- Rayleigh's method and Buckingham's  $\pi$  – theorem.
- Model Analysis and 3 types of similarities.
- 5 types of forces and 5 dimensionless numbers.
- 2 model laws and scale ratio of different quantities.
- Near about 30 formulae's.