NPR COLLEGE OF ENGINEERING & TECHNOLOGY

Subject : DYNAMICS OF MACHINERY

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NPRCET

UNIT 4

FORCED VIBRATION

DYNAMICS OF MACHINERY

FORCED VIBRATION

Define Forced Vibration.

Forced Vibration is the vibration of a body under the influence of an external force. When an external force is acting, the body does not vibrate with its own natural frequency, but vibrates with the frequency of the applied external force. Ex: Air compressors, IC engines, Machine Tool.

Mention the types of forced vibrations forcing functions.

(a) Periodic forcing functions;

(b) Impulsive forcing functions;

(c) Random forcing functions.

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FORCED VIBRATION

Distinguish between impulsive forcing function and random forcing function.

Impulsive forcing functions are non periodic and produce transient vibrations. They are quite common, but die out soon and hence not important. Ex: Rock explosion, punching.

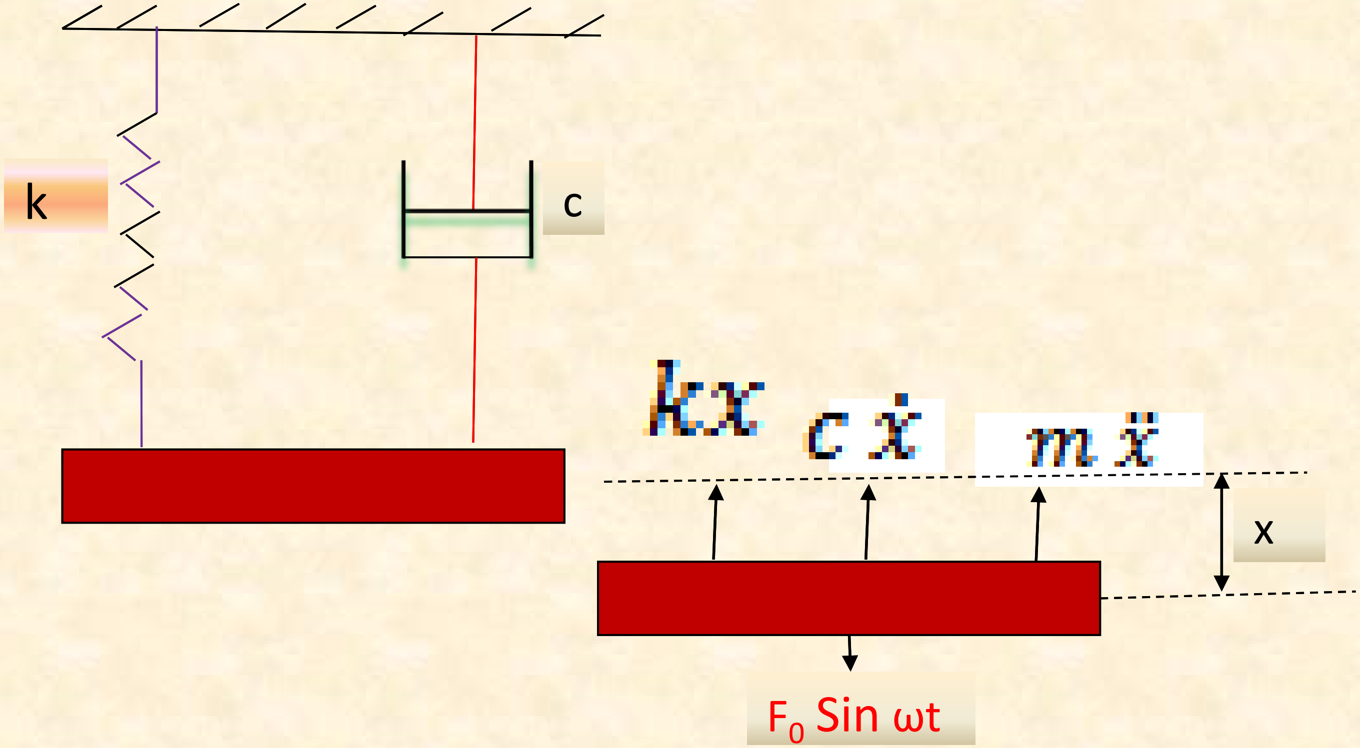
Random forcing functions are uncommon, unpredictable, and non deterministic. They produce random vibrations. Ex: Ground motion during earth quake.

What are the types of forced vibrations?

(a) Damped vibrations (Damping is provided)

(b) Undamped vibrations (No damping is provided C = 0)

Forced Vibration with Harmonic Forcing



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HARMONIC FORCING

Mention the equation of motion of spring mass system with a harmonic force,

$F = F_o \cos \omega t$

Equation of motion is given by, $m \ddot{x} + c \dot{x} + k x = F_o \cos \omega t$

Where $(m \ddot{x})$ is inertia force; $(c \dot{x})$ is damping force; and $(k x)$ is spring force; and $(F_o \cos \omega t)$ is the excitation force.

Explain the terms transient vibration and steady state vibration.

The equation of displacement is given by,

$$x = X^{-\xi \omega_n t} \sin(\omega_d t + \phi) + \frac{F_0 \sin(\omega_n t - \phi)}{(k - m\omega^2)^2 + (c\omega)^2}$$

The first part of the solution decays with time and vanishes ultimately. So it is called transient vibration.

The second part is a particular integral and is a sinusoidal vibration with constant amplitude. Hence it is called steady state vibration.

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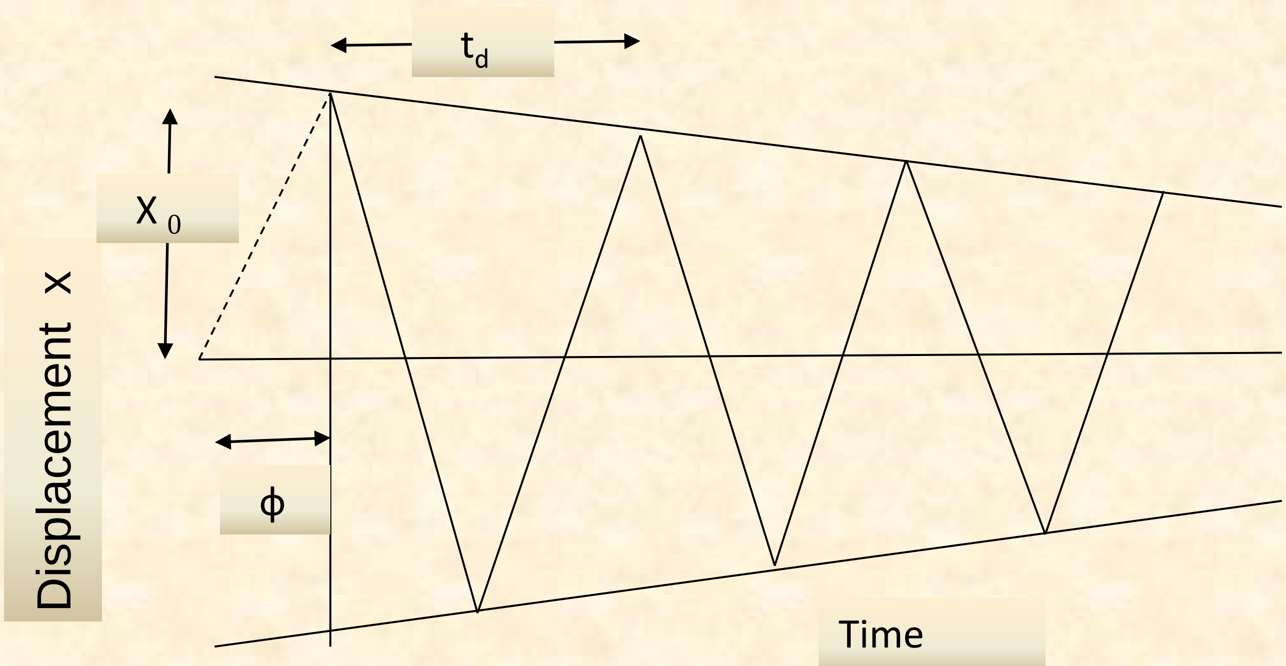
UNDER DAMPED VIBRATION



Displacement vs Time Plot for under damped system

DYNAMICS OF MACHINERY

HARMONIC FORCING

Amplitude (Max. displacement) of forced vibration

(OR)

Amplitude of steady state response

$$A = \frac{F_0}{\sqrt{[(k - m\,\omega^2)^2 + (c\,\omega)^2]}} = \frac{(F_0/k)}{\sqrt{[(1 - (\omega/\omega_n)^2)^2 + (2\,\xi\omega/\omega_n)^2]}}$$

$$\text{Phase Lag},\ \phi = \tan^{-1} \frac{(2\,\xi\omega/\omega_n)}{(1 - (\omega/\omega_n)^2)}$$

DYNAMICS OF MACHINERY

HARMONIC FORCING

Define the dynamic magnifier or magnification factor or magnification ratio.

Dynamic magnifier is the ratio of amplitude of steady state response to zero frequency deflection.

Zero frequency deflection is the deflection under the action of impressed force.

Since $X_0 = F_0/k$, also called static deflection

A is also referred as X_{max}

$$MF = \frac{A}{X_0} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega n}\right)\right]^2}}$$

DYNAMICS OF MACHINERY

RESPONSE TO PERIODIC FORCING

Show that amplitude of forced vibrations due to unbalanced rotating mass is constant at high speeds.

At high speeds the dimensionless amplitude tends to unity. Hence the result,

$A / [m_o\, e / m) = 1$; $A = m_o\, e / m$

The amplitude is constant and it is independent of frequency and damping in the system.

Note: m_o is reciprocating mass balanced in kg (i.e. unbalanced rotating mass in kg); e is eccentricity in m; and m is total mass of the engine;

A is amplitude in m

DYNAMICS OF MACHINERY

FORCED VIBRATION CAUSED BY UNBALANCE

How does unbalanced rotating mass cause forced vibration?

For example, electric motor, turbine, other rotating machineries have some amount of unbalance left in them even after rectifying their unbalance on precision balancing machines. This unbalanced rotating mass produces centrifugal force which acts as exciting force and causes forced vibrations of the machine.

Why is the radius of crank taken as eccentricity in case of forced vibrations due to reciprocating unbalance?

Some amount of unbalanced reciprocating mass in a machine causes forced vibrations of the machine. As the stroke is proportional to radius of crank, the radius of crank is taken as eccentricity of reciprocating mass. This unbalanced reciprocating mass produces inertia force which acts as exciting force and causes forced vibration of the machine.

DYNAMICS OF MACHINERY

SUPPORT MOTION

What is base excitation or support motion?

The support itself undergoes excitation with some displacement. For example, consider the movement of automobile trailer or car on wavy road. In this case as the road is sinusoidal in shape, it is considered that the road itself produces an excitation. This excitation of support produces a force which acts as exciting force on the system. Hence the system undergoes forced vibration. This is called forced vibration due to excitation of support.

DYNAMICS OF MACHINERY

SUPPORT MOTION- AMPLITUDE TRANSMISSIBILITY

Forced vibration due to support excitation: The governing equation is

Given by, $m\ddot{x} + c\dot{x} + kx = Y\sqrt{k^2 + (c\omega)^2}\ \sin(\omega t+\alpha)$

Where $(m\ddot{x})$ is inertia force; $(c\dot{x})$ is damping force; and (kx) is spring force; and

$Y\sqrt{k^2 + (c\omega)^2}\ \sin(\omega t+\alpha)$ is the support excitation force.

$$\text{Amplitude of vibration } A = \frac{Y\sqrt{k^2 + (c\omega)^2}}{(k - m\omega^2)^2 + (c\omega)^2}$$

Displacement transmission ratio (OR) Amplitude ratio

$$\frac{A}{Y} = \frac{\sqrt{[1 + (2\xi\omega/\omega_n)^2]}}{\sqrt{[(1 - (\omega/\omega_n)^2)^2 + (2\xi\omega/\omega_n)^2]}}$$

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FORCE TRANSMISSIBILITY

Define transmissibility or isolation factor.

It is the ratio of force transmitted to the foundation F_T to the force applied F_o

$$" \quad \varepsilon = F_T / F_o \quad "$$

Mention the angle by which the transmitted force lags the impressed force.

Angle of lag

$$(\phi - \alpha) = \psi = \tan^{-1}\left[\frac{(2\xi\,\omega/\omega_n)}{[(1-(\omega/\omega_n)^2]}\right] - \tan^{-1}(2\xi\,\omega/\omega_n)$$

DYNAMICS OF MACHINERY

FORCE TRANSMISSIBILITY

What will happen when dampers are used for frequency ratio greater than √2?

When $(\omega/\omega_n) > \sqrt{2}$, transmissibility increases as damping is increased. Hence dampers should not be used in this range.

Mention the equation of phase angle between applied force and displacement.

$$\phi = \tan^{-1}\left[\frac{(2\xi\, \omega/\omega_n)}{[(1 - (\omega/\omega_n)^2]}\right]$$

DYNAMICS OF MACHINERY

VIBRATION ISOLATION

Define vibration isolation.

The process of reducing the vibrations of machines (and hence reducing the transmitted force to the foundation) using vibration isolating materials such as cork, rubber is called vibration isolation.

Mention the materials used for vibration isolation.

Springs, Dampers, Pads of rubber, cork etc.

What is the importance of vibration isolation?

Vibrations are produced in machines having unbalanced mass. For example, inertia force developed in a reciprocating engine or centrifugal force developed in a rotating machinery causes vibrations. These vibrations will be transmitted to the foundation upon which the machines are mounted. This is undesirable and should be avoided. Hence vibrations should be eliminated or at least should be reduced using vibration isolators.

Governing Equations for Harmonic Motion

$$m\,\ddot{x} + c\,\dot{x} + k\,x = F_0\,Sin\,\omega t$$

$$x = X\,e^{-\xi\omega_n t}\sin(\omega_0 t + \phi) + \frac{Fo\,Sin\,(\omega t - \phi)}{(k - m\omega^2)^2 + (c\omega)^2}$$

$$A = \frac{F_0/_k}{\sqrt{\left[1 - \left(\frac{\omega}{\omega n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega n}\right)\right]^2}}$$

$$Tan\,\phi = \frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]}$$

$$MF = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega n}\right)\right]^2}}$$

Governing Equations for rotating /reciprocating unbalance

$$A = \frac{m_r e \omega^2 / k}{\sqrt{\left[1-\left(\frac{\omega}{\omega n}\right)^2\right]^2+\left[2\xi\left(\frac{\omega}{\omega n}\right)\right]^2}}$$

$$Tan\,\phi = \frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]}$$

$$\frac{A}{m_r e/m} = \frac{\left(\omega/\omega_n\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega n}\right)^2\right]^2+\left[2\xi\left(\frac{\omega}{\omega n}\right)\right]^2}}$$

Governing Equations for Support Vibrations

$$m\,\ddot{x} + c\,\dot{x} + k\,x = Y\sqrt{k^2 + (c\omega)^2}\,Sin\,\omega t$$

$$\frac{A}{Y} = \frac{\sqrt{1 + [2\,\xi(\frac{\omega}{\omega n})]^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega n}\right)\right]^2}}$$

Angle of Lag = $\psi = \Phi - \alpha = Tan^{-1}\dfrac{2\,\xi(\frac{\omega}{\omega n})}{1-\left(\frac{\omega}{\omega n}\right)^2}\;\; Tan^{-1}[2\,\xi(\frac{\omega}{\omega n})]$

DYNAMICS OF MACHINERY

Given: Helical spring; Fixed at one end and mass at the other end; Mass m = 10 kg;
Stiffness of spring k = 10 N/mm;
Decrease in amplitude in 4 complete revolutions = (1/10) initial value;
Periodic force on mass in vertical direction = 150 cos 50 t N
To find: (a) Amplitude of forced vibration; (b) Amplitude value of resonance

$$m = 10\,kg;\quad k = 10\,N/mm = 10^4\,N/mm;\quad F_t = F_0 \,Cos\,\omega t = 150\;Cos\,\omega t;\qquad \frac{X_0}{10} = X_4$$

$$\omega_n = \sqrt{k/m} = 31.62\,rad/s \qquad\qquad \delta = \ln\left(\frac{X_0}{X_1}\right) = \frac{1}{4}\,ln\left(\frac{X_0}{X_4}\right) = 0.5756$$

$$c_c = 2m\omega_n = 632.46\;Ns/m$$

$$\xi = \frac{c}{c_c} = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} = 0.091$$

$$\omega = \omega n\sqrt{(1-\xi^2)} = 31.49\;rad/s$$

$$\frac{A}{(F_0/k)} = \frac{1}{\sqrt{\left[1-(\frac{\omega}{\omega_n})^2\right]^2 + \left[2\xi(\frac{\omega}{\omega_n})\right]^2}} = 5.499$$ A = 0.0827 m

At resonance, $\frac{A}{(F_0/k)} = \frac{1}{2\xi} = 5.483$ A = 0.0824 m

;

DYNAMICS OF MACHINERY

Given: Mass suspended from a spring; Mass of body m = 15 kg;
Static deflection δ = 0.012 m;
To find: (a) frequency of free vibration;
(b) Viscous damping force if motion is periodic with speed 1 mm/c;
(c) Amplitude of ultimate motion if maximum value of disturbing force is 100 N and frequency of vibration is 6 Hz.

$\omega_n = \sqrt{\frac{g}{\delta}}$ =28.59 rad/s $\quad$ $f_n = \frac{\omega_n}{2\pi} = \frac{1}{t_p}$ =4.55 Hz

Damping force required to make the motion aperiodic at a speed of 1 mm/s:
Motion is aperiodic when frequency $f_n = 0$ or when it is critically damped $\omega=\omega_n$

‘ $\omega = \omega_n$ = 28.59 rad/s $\quad$ $C = c_c = 2\ m\ \omega_n$ = 857 Ns/m = 0.857 Ns/mm

Damping force required to make the motion aperiodic at a speed of 1 mm/s = 0.857 N

Amplitude of forced vibration A or x_{max} : For forced vibration, ω = 2 п f = 2 п 6 = 37.7 rad/s

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}}$ $\quad$ Hence stiffness of spring k = mg/δ = 37.7 rad/s

Frequency ratio ω/ωn = 37.7/28.59 = 1.319 $\quad$ $A = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega n}\right)\right]^2}}$

Amplitude of forced vibration A or x_{max} = 0.0029 m

DYNAMICS OF MACHINERY

Given: Single cylinder vertical petrol engine; Mass of engine m = 200 kg;
Mass mounted on steel chassis frame;
Static deflection of frame δ = 0.0024 m;
Mass of reciprocating parts m_R = 9 kg; vertical stroke of SHM = 0.16 m;
Dash pot is used; Damping coefficient C = 1 Ns/mm;
Speed of driving shaft N = 500 rpm;
To find: (a) Amplitude of forced vibration under steady state; (b) Speed of driving shaft at which resonance will occur.

$$m = 200\,kg; m_R = 9\,kg;\ \delta = 0.0024\,m; c = 1\,\frac{Ns}{mm} = 1000\,\frac{Ns}{m};$$

$$L = 0.16\,m;\ r = e = \frac{L}{2} = 0.08\,m;\ N = 500\,rpm;\ \omega = \frac{2\pi N}{60} = 52.38\,\frac{rad}{s};$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}} = 63.93\,\frac{rad}{s}; \qquad c_c = 2m\omega_n = 2\,x\,200\,x\,63.93 = 25572\,Ns/m$$

$$\xi = \frac{c}{c_c} = \frac{1000}{25572} = 0.0391$$

$$\frac{A}{(m_r\,e/m)} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-(\frac{\omega}{\omega_n})^2\right]^2+\left[2\xi(\frac{\omega}{\omega_n})\right]^2}} = 2.005$$

Hence A = 0.0072 m

$$\omega_n = \frac{2\pi N_{res}}{60} = 63.93\,rad/s\,;\,hence\ N_{res} = 610.5\,rpm$$

DYNAMICS OF MACHINERY

Given: Mass of machine m = 75 kg; Stiffness of spring k = 1200 kN/m;
Damping factor = 0.2; Mass of reciprocating piston m_R = 2 kg;
Stroke length L = 0.08 m; Speed = 3000 cycles/min
Motion is SHM.
To find: (a) Amplitude of motion of machine;
(b) Phase angle with respect to exciting force;
(c) Force transmitted to foundation;
(d) Phase angle of transmitted force with respect to exciting force;

$m = 75\,kg;\;\; k = 12\,x10^5\,N/mm;\;\; \xi = \frac{c}{cc} = 0.2;\quad N = 3000\,rpm;\quad m_R = 2\,kg;\; L = 0.08\,m$

$\omega_n = \sqrt{k/m} = 126.49\,rad/s \qquad \omega = \frac{2\pi N}{60} = 314.16\,rad/s$

' e = L/2 = 0.04 m

Unbalanced force $F_0 = m_R\,\omega^2\,e$ = 7895.68 N

$$A = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega n}\right)\right]^2}}$$

Phase angle $\Phi = tan^{-1}\left(\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]}\right) = 169.12^0$

A = X max = 0.00125 m

$$\frac{F_T}{F_0} = \sqrt{\frac{1+\left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

F_T = 2112.62 N

Phase angle of transmitted force $\alpha = tan^{-1}\left(2\xi\left(\frac{\omega}{\omega n}\right)\right) = 44.8^o$

DYNAMICS OF MACHINERY

Given: Mass of machine m = 1000 kg;
External Force F = 2450 N; Frequency of external force = 1500 rpm;
Isolator used is Rubber;
Static deflection of isolator δ = 0.002 m; Estimated damping ξ= 0.2;
To find: (a) Force transmitted to foundation; (b) Amplitude of vibration; and (c) phase angle

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}} = 70\frac{rad}{s};$$

K = mg/δ = 4905000 N/m; ω = 2πN/60 = 157.14 rad/s; ω/ω_n = 2.244

$$c_c = 2m\omega_n = 2\,x\,1000\,x\,70 = 140000\ Ns/m$$

$$\xi = \frac{c}{c_c} = 0.2\,;\ hence\ c = $$ 28000 Ns/m

$$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1+\left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^{2+}+\left(2\xi(\omega/\omega_n)\right)^2}}$$ = 0.325; F_T = 798.8 N

$$\frac{A}{X_0} = \frac{1}{\sqrt{[1-(\omega/\omega_n)^2]^{2+}+\left(2\xi(\omega/\omega_n)\right)^2}}$$

A = 1.2 x 10 $^{-4}$ m

$$\Phi = tan^{-1}\left(\frac{2\xi(\frac{\omega}{\omega_n})}{\left[1-(\frac{\omega}{\omega_n})^2\right]}\right) = 167^0 25'$$

DYNAMICS OF MACHINERY

Given: Mass of refrigerator unit m = 30 kg; No. of support springs n =3;
Force transmitted to support structure = 0.1 impressed force
Speed of the unit = 420 rpm
To find: (a) Stiffness of each spring;
(b) Deduce the expression for transmissibility.

‘m = 30 kg; n = 3; $F_T = 0.1\ F_o$; N = 420 rpm;

‘ $\omega = 2\pi N/60 = 43.98$ rad/s;

‘ $\varepsilon = F_T / F_O = 0.1$

No damper is used → $\xi = 0$

$$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1 + \left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1 - (\omega/\omega_n)^2]^{2+} + \left(2\xi(\omega/\omega_n)\right)^2}}$$

$$\varepsilon = \frac{F_T}{F_o} = \frac{1}{\sqrt{[1 - (\omega/\omega_n)^2]^2}}$$

= 0.1; hence $\omega/\omega_n = 3.32$; $\omega_n = 13.26$ rad/s

$$\omega_n = \sqrt{k_{eq}/m} = 13.26\ rad/s$$

Hence, $k_{eq} = 5275.2$ N/m; n = 3 in parallel
For each spring, k = keq/3 = 1758.4 N/m

DYNAMICS OF MACHINERY

Given: Mass of machine m = 80 kg; No. of support springs n = 4;
Mass of reciprocating parts m_R = 2.2 kg; Vertical stroke of SHM L = 0.1 m;
Neglect damping; Force transmitted to foundation = (1/20) impressed force;
Speed of machine crank shaft = 800 rpm;
Damping reduces amplitudes of successive vibrations by 30%;
To find: (a) Combined stiffness of spring; (b) Force transmitted to the foundation at 800 rpm;
(c) Force transmitted to the foundation at resonance; and
(d) Amplitude of vibration at resonance.

‘m =80 kg; m_R = 2.2 kg; n = 4; $X_1 = 0.7\ X_0$; N = 800 rpm; $F_T = F_o / 20$;
L = 0.1 m; e = L/2 = 0.05 m

‘ ω = 2πN/60 = 83.81 rad/s;

No damper is used → ξ = 0

$$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1 + \left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^{2+} + \left(2\xi(\omega/\omega_n)\right)^2}}$$

$$\varepsilon = \frac{F_T}{F_o} = \frac{1}{\sqrt{[1-(\omega/\omega_n)^2]^2}}$$

= 0.1; hence ω/ω_n= 4.583; ω_n = 18.287rad/s

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{g}{\delta}} = 18.287\frac{rad}{s};$$

Hence, k_{eq} = 26739 N/m; n = 4 in parallel
For each spring, k_1 = keq/4= 6684.8 N/m

Logarithmic decrement δ = ln (X_0/X_1) = ln(1/0.7) = 0.3567

Contd.

Given: Mass of machine m = 80 kg;   No. of support springs n = 4;

Mass of reciprocating parts m_R = 2.2 kg; Vertical stroke of SHM   L = 0.1 m;

Neglect damping;   Force transmitted to foundation = (1/20) impressed force;

Speed of machine crank shaft = 800 rpm;

Damping reduces amplitudes of successive vibrations by 30%;

To find: (a) Combined stiffness of spring; (b) Force transmitted to the foundation at 800 rpm;

(c) Force transmitted to the foundation at resonance; and

(d) Amplitude of vibration at resonance.

$$\delta = \frac{2\pi\xi}{\sqrt{(1-\xi^2)}}\ ;\ \xi = \frac{c}{c_c} = \sqrt{\frac{\delta^2}{4\pi^2+\delta^2}} = 0.067$$

$F_o = m_R\, \omega^2\, e = 772.63$ N

$$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1+\left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^{2+}+\left(2\xi(\omega/\omega_n)\right)^2}} = 0.0586;\quad F_T = 0.0586\text{x}772.63 = 45.3 \text{ N}$$

At resonance, $\omega = \omega_n$ = 18.28 rad/s; $F_o = m_R\, \omega^2\, e = 2.2\text{x}18.28^2 \text{ x } 0.05 = 36.75$ N

At resonance, $$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1+\left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^{2+}+\left(2\xi(\omega/\omega_n)\right)^2}} = 7.53;\quad F_T = 276.73 \text{ N}$$

At resonance, $$\frac{A}{(F_0/k_{eq})} = \frac{1}{2\xi} = 7.463$$ A = 7.463x772.1/26739 = 0.0.01 m

DYNAMICS OF MACHINERY

Given: Mass of machine m = 100 kg; Spring stiffness k = 700 kN/m = 700000 N/m;
Disturbing force due to unbalanced rotating parts = 350 N;
Speed N = 3000 rpm;   Damping factor   ξ = 0.2
To find: (a) Amplitude of motion due to unbalance;
(b) Transmissibility; and   (c) Transmitted force.

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{g}{\delta}} \qquad \omega = 2\pi N/60$$

$\omega/\omega_n =$

$$\frac{A}{F_o/k} = \frac{1}{\sqrt{[1-(\omega/\omega_n)^2]^{2+} + \left(2\xi(\omega/\omega_n)\right)^2}}$$

A =

$$\Phi = tan^{-1}\left(\frac{2\xi(\frac{\omega}{\omega_n})}{\left[1-(\frac{\omega}{\omega_n})^2\right]}\right) = 167^{0}25'$$

$$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1+\left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^{2+} + \left(2\xi(\omega/\omega_n)\right)^2}}$$

F_T = 4789 N

DYNAMICS OF MACHINERY

Given: Electric motor supported on springs and dashpot;
Spring stiffness k = 6400 N/m; Dashpot resistance = 500 N at 4 m/s;
Unbalanced mass = 0.5 kg; Radius of unbalanced mass r = 0.05 m;
Total mass of vibratory system = 20 kg; Speed of motor N = 400 rpm;
To find: (a) Damping factor; (b) Amplitude of vibration; (c) Phase angle;
(d) Resonant speed; (e) Resonant amplitude; and
(f) Force exerted by spring and dashpot on the motor.

‘ c = F/v = 500/4 = 125 Ns/m; $c_c = 2m\,\omega n = 2x20x\ 17.885 = 715.5418$ Ns/m;
$\xi = c/\ c_c = 0.175$

$$\frac{A}{(m_r\,e/m)} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-(\frac{\omega}{\omega_n})^2\right]^2+\left[2\xi(\frac{\omega}{\omega_n})\right]^2}} =$$

A = 0.0015 m

$$Tan\ \phi = \frac{2\xi\,(\frac{\omega}{\omega_n})}{[1-\left(\frac{\omega}{\omega_n}\right)^2]}$$

At resonance, $\frac{A}{(m_r\,e/m)} = \frac{1}{2\xi} = \frac{1}{2x0.175} = 2.8571$

$A_{res} = 0.00357$ m

‘ $\omega = 2\pi N/60 = 41.905$ rad/s; $F_C = c\omega A = 125x41.905x0.0015 = 7.86$ N

$F_S = kA = 6400 \times 0.0015 = 9.60$ N; $F = \sqrt{F_C{}^2 + F_s{}^2}$ = 12.4 N

DYNAMICS OF MACHINERY

Given: Mass of electric motor = 120 kg; Speed of motor N = 1500 rpm;
Armature mass = 35 kg;
CG of armature mass lies 0.5 mm from the axis of rotation;
Number of support springs n = 5; Damping is negligible;
Force transmitted = (1/11) impressed force
To find: (a) Stiffness of each spring; (b) Natural frequency of the system; and
(c) Dynamic force transmitted to the base at the operating speed.

‘ m = 150 kg; N = 1500 rpm→ $\omega = 2\pi N/60 = 157.08$ rad/s; e = 0.5 mm = 0.0005 m;
$m_o = 35$ kg; $\varepsilon = F_T/F_o = 1/11$

Total mass m = 150+35 = 185 k

When damping is negligible, $\xi = 0$;

$$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1 + \left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1 - (\omega/\omega_n)^2]^{2+} + \left(2\xi(\omega/\omega_n)\right)^2}}$$

$$\frac{\omega}{\omega_n} = \sqrt{1 - \xi^2} = 3.464$$

$$\omega_n = \sqrt{\frac{k}{m}} = 45.34\, rad/s$$

‘ k= 32480.7 N/m

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{t} = 7.22\, Hz$$

Fo = mo ω^2 e = 35x157.08 ^{2}x0.0005 = 431.7972 N

FT = ε Fo = (1/11)431.7972 = 39.25 N

DYNAMICS OF MACHINERY

Given: Number of isolators = 6; Stiffness of each isolator k = 32000 N/m;
Number of dashpots =6; Damping resistance = 400 Ns/m (?)
Mass of rotating device m = 30 kg; Rotating speed = 600 rpm;
Amplitude of vibration $Y = X_o = 0.06$ m;
To find: (a) Amplitude of vibration of body;
(b) Dynamic load on each isolator.

For support vibration, $$\frac{A}{Y} = \frac{\sqrt{1+\left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^{2+}+\left(2\xi(\omega/\omega_n)\right)^2}}$$

Check with key 10417 14b

A/Y = 1.4549; A = 0.06x1.4549 = 0.087 m

Dynamic load on each isolator:

' $\omega = 2\pi N/60 = 62.86$ rad/s; $F_C = c\omega A$ = 400x62.86x0.087= 2187.528 N

$F_S = kA$ = 32000 x 0.087 = 2784 N; $F = \sqrt{F_C^2 + F_s^2}$ = 3540.6 N

DYNAMICS OF MACHINERY

Given: Refrigerator mass =40 kg; Number of support springs =3;
Speed = 520 rpm; % of shaking force allowed to be transmitted to support structure = 12%
To find: Stiffness of each spring k

, m = 40 kg; N = 520 rpm →ω = 2πN/60=54.45 rad/s; c = 0 since no damping, hence $\xi = c/c_c = 0$; ε = 0.12

$$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1 + \left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^{2+} + \left(2\xi(\omega/\omega_n)\right)^2}} = 0.12$$

Solving, $\omega/\omega_n = 3.056$; $\omega_n = 17.82$ rad/s;

$$\omega_n = \sqrt{k_{eq}/m} = 17.82\ rad/s$$

K eq = 12706.3 N/m
3 springs are parallel →k eq = k + k + k
Hence for each spring, k = k eq/3 = 4235.4 N/m

DYNAMICS OF MACHINERY

Given: Machine mass = 100 kg; Supported on spring;
Total stiffness k = 7.84 x 10^5 N/m;
Unbalanced rotating masses cause disturbing force 392 N at a speed 3000 rpm;
Damping factor =0.2;
To find: (a) Amplitude of motion due to unbalance;
(b) Transmissibility; and (c) Force transmitted.

, m = 100 kg; k =7.84x10^5 N/m; F0 = m ω2 e = 392 N;
N = 3000 rpm →ω = 2πN/60=314.16 rad/s; ξ = 0.2

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{7.84x10^5}{100}} = 88.5\, rad/s$$

$$\frac{A}{(m_r\, e/m)} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-(\frac{\omega}{\omega_n})^2\right]^2 + \left[2\xi(\frac{\omega}{\omega_n})\right]^2}}$$

A = 4.3x10^{-5} m

$$\varepsilon = \frac{F_T}{F_o} = \frac{\sqrt{1+\left(2\xi(\omega/\omega_n)\right)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^{2+} + \left(2\xi(\omega/\omega_n)\right)^2}}$$

ε = 0.149

$F_T = F_0\, ε$ = 392x0.149 = 58.58 N

DYNAMICS OF MACHINERY

DYNAMICS OF MACHINERY