UNIT-V MECHNISMS FOR CONTROL

Lecture-1

Types of Mechanisms for control are

1) Governors.
2) Gyroscope.

Governors:

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

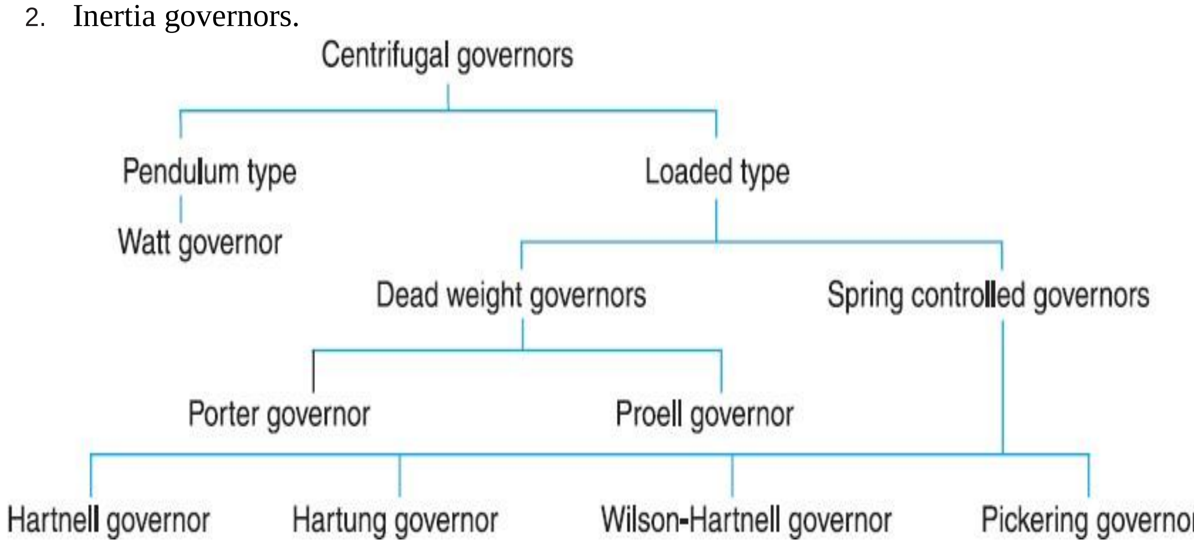
The function of a flywheel in an engine is entirely different from that of a governor. It controls the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation. It does not control the speed variations caused by a varying load. The varying demand for power is met by the governor regulating the supply of working fluid.

Difference between a Flywheel and Governor:

Types of Governors

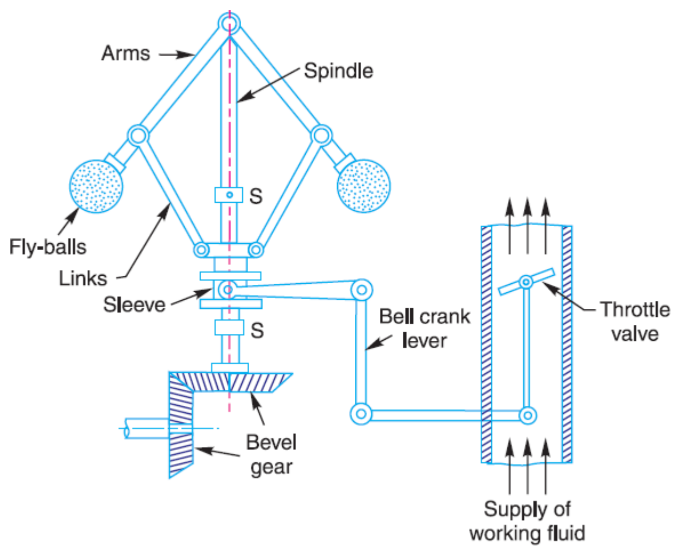
The governors may, broadly, be classified as

1. Centrifugal governors
2. Inertia governors.



Centrifugal governors:

The centrifugal governors are based on the balancing of centrifugal force on the rotating ballsby an equal and opposite radial force, known as the controlling force.It consists of two balls ofequal mass, which are attached to the arms as shown in Fig. These balls are known as governorballs or fly balls.



The balls revolve with a spindle, whichis driven by the engine through bevel gears. The upperends of the arms are pivoted to the spindle, so that theballs may rise up or fall down as they revolve about thevertical axis. The arms are connected by the links to asleeve, which is keyed to the spindle. This sleeve revolveswith the spindle; but can slide up and down.The balls and the sleeve risewhen the spindle speedincreases, and falls when the speed decreases. In orderto limit the travel of the sleeve in upward and downwarddirections, two stops S, S are provided on thespindle. The sleeve isconnected by a bell crank leverto a throttle valve. The supply of the working fluid decreaseswhen the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.

Lecture-2

Terms Used in Governors

1. Height of a governor. It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by h.
2. Equilibrium speed. It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.
3. Mean equilibrium speed. It is the speed at the mean position of the balls or the sleeve.
4. Maximum and minimum equilibrium speeds. The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

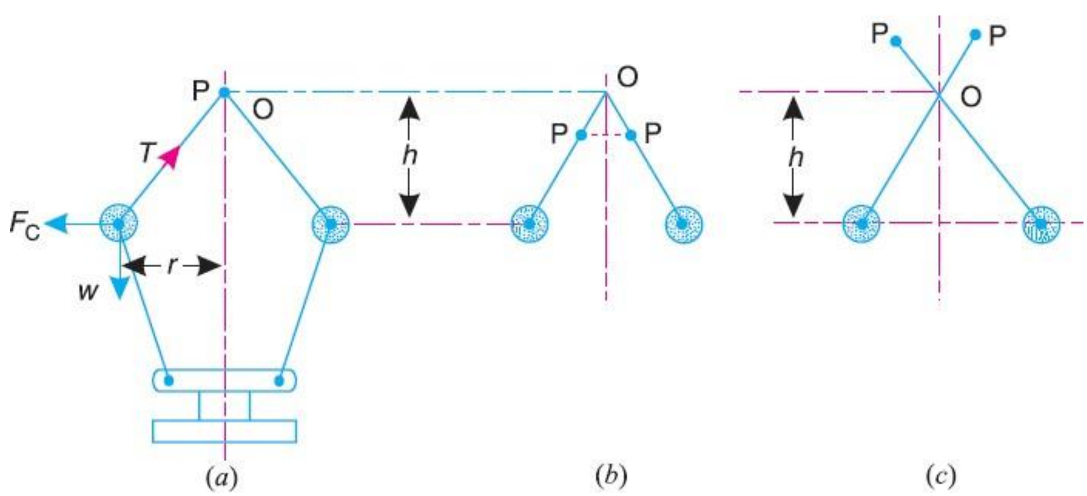
Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the minimum equilibrium speeds.

5. Sleeve lift. It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Watt Governor:

The simplest form of a centrifugal governor is a Watt governor, as shown in Fig. It isbasically a conical pendulum with links attached to a sleeve of negligible mass. The arms of thegovernor may be connected to the spindle in the following three ways:

1. The pivot P may be on the spindle axis as shown in Fig. (a).
2. The pivot P may be offset from the spindle axis and the arms when produced intersect at O, as shown in Fig. (b).
3. The pivot P may be offset, but the arms cross the axis at O, as shown in Fig.(c).



m = Mass of the ball in kg,
w = Weight of the ball in newtons = $m.g$,
T = Tension in the arm in newtons,
ξ = Angular velocity of the arm and ball about the spindle axis in rad/s,
r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,
F_C = Centrifugal force acting on the ball in newtons = $m. \xi^2.r$, and
h = Height of the governor in metres.

$$h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres}$$

Problem-1

Calculate the vertical height of a Watt governor when it rotates at 60 r.p.m. Also find the change in vertical height when its speed increases to 61 r.p.m.

Given: N_1 = 60 r.p.m. ; N_2 = 61 r.p.m.

Initial height

We know that initial height,

$$h_1 = \frac{895}{(N_1)^2} = \frac{895}{(60)^2} = 0.248 \text{ m}$$

Change in vertical height

We know that final height,

$$h_2 = \frac{895}{(N_2)^2} = \frac{895}{(61)^2} = 0.24 \text{ m}$$

Change in vertical height

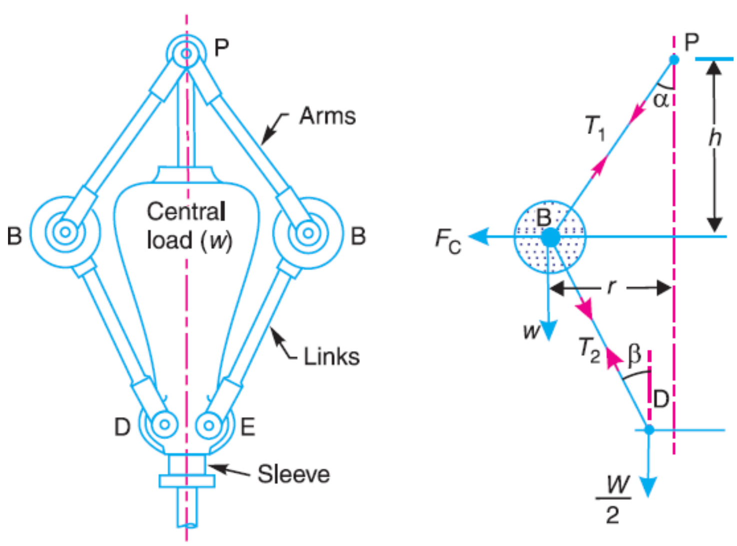
$= h_1 - h_2 = 0.248 - 0.24 = 0.008$ m = **8 mm**

Lecture-3

Porter Governor:

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig.(a). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Consider the forces acting on one-half of the governor as shown in Fig.(b).



(a) (b)

m = Mass of each ball in kg,

w = Weight of each ball in newtons = $m.g$,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = $M.g$,

r = Radius of rotation in metres,

h = Height of governor in metres,

N = Speed of the balls in r.p.m.

ξ = Angular speed of the balls in rad/s = $2\pi N/60$ rad/s,

F_C = Centrifugal force acting on the ball in newtons = $m.\xi^2.r$,

T_1 = Force in the arm in newtons,

T_2 = Force in the link in newtons,

α = Angle of inclination of the arm (or upper link) to the vertical, and

⌡ = Angle of inclination of the link (or lower link) to the vertical.

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

1. When the length of arms are equal to the length of links and the points P and D lie on the same vertical line, then
$\tan\alpha = \tan\beta$ or $q = \tan\alpha / \tan\beta = 1$

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}$$

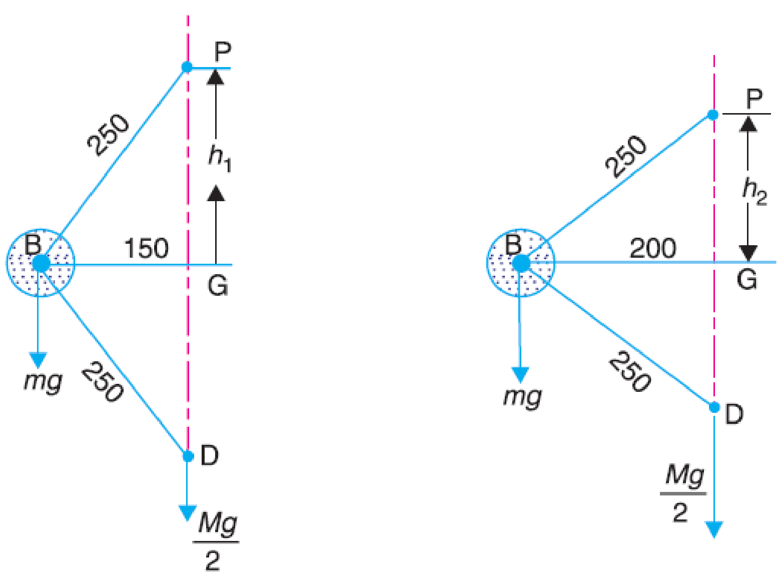
2. When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.
If F = Frictional force acting on the sleeve in newtons, then the equations (v) and (vi) may be written as

$$N^2 = \frac{m.g + \left(\frac{M.g \pm F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

The + sign is used when the sleeve moves upwards or the governor speed increases and negative sign is used when the sleeve moves downwards or the governor speed decreases.
Problem – 2

A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. Theradius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.
(AU 2006)

Gn :$BP = BD$ = 250 mm = 0.25 m ; m = 5 kg ; M = 15 kg ; r_1 = 150 mm = 0.15m; r_2 = 200 mm = 0.2 m



(*a*) Minimum position. (*b*) Maximum position.

Minimum speed when r_1 = BG = 0.15 m
Let $N1$ = Minimum speed.
From Fig(*a*), we find that height of the governor,

$h = PG = \sqrt{(PB)^2 - (BG)^2} = 0.2$ m

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+15}{5} \times \frac{895}{0.2} = 17\,900$$

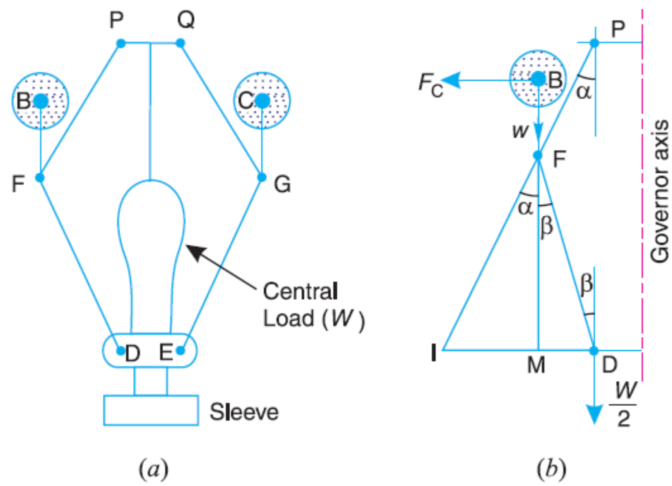
N_1 = 133.8 r.p.m.

Maximum speed when r_2 = BG = 0.2 m

Let N_2 = Maximum speed.

From Fig(*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.2)^2} = 0.15 \text{ m}$$



$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+15}{5} \times \frac{895}{0.15} = 23\,867$$

N_2 = 154.5 r.p.m

Range of speed

We know that range of speed= $N_2 - N_1$ = 154.4 – 133.8 = **20.7 r.p.m.**

Lecture-4

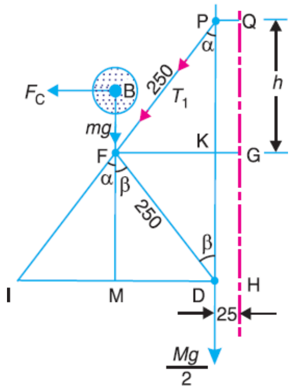
Proell Governor

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig (*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively. Consider the equilibrium of the forces on one-half of the governor as shown in Fig (*b*). The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicular to the spindle axis. The perpendicular *BM* is drawn on *ID*.

Problem – 3

A governor of the Proell type has each arm 250 mm long. The pivots of the upper and lower arms are 25 mm from the axis. The central load acting on the sleeve has a mass of 25 kg and the each rotating ball has a mass of 3.2 kg. When the governor sleeve is in mid-position, the extension link of the lower arm is vertical and the radius of the path of rotation of the masses is175 mm. The vertical height of the governor is 200 mm.If the governor speed is 160 r.p.m. when in mid-position, find : 1. length of the extensionlink; and 2. tension in the upper arm.

Given: $PF = DF = 250$ mm $= 0.25$ m ; $PQ = DH = KG = 25$ mm $= 0.025$ m ; $M = 25$ kg ; $m = 3.2$ kg ; $r = FG = 175$ mm $= 0.175$ m ; $h = QG = PK = 200$ mm $= 0.2$ m ; $N = 160$ r.p.m.



$$N^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h} \qquad \ldots (\because\ \alpha = \beta \text{ or } q = 1)$$

$$\frac{(1+q)}{} \Bigg] \frac{895}{h}$$

$$(160)^2 = \frac{0.2}{BM}\left(\frac{3.2+25}{3.2}\right)\frac{895}{0.2} = \frac{7887}{BM}$$

$$BM = 7887/(160)^2 = 0.308 \text{ m}$$

$BF = BM - FM = 0.308 - 0.2 = 0.108$ m $= 108$ mm

Tension in the upper arm

$$T_1 = \text{Tension in the upper arm.}$$

$$PK = \sqrt{(PF)^2 - (FK)^2} = \sqrt{(PF)^2 - (FG - KG)^2}$$

$$= \sqrt{(250)^2 - (175-25)^2} = 200 \text{ mm}$$

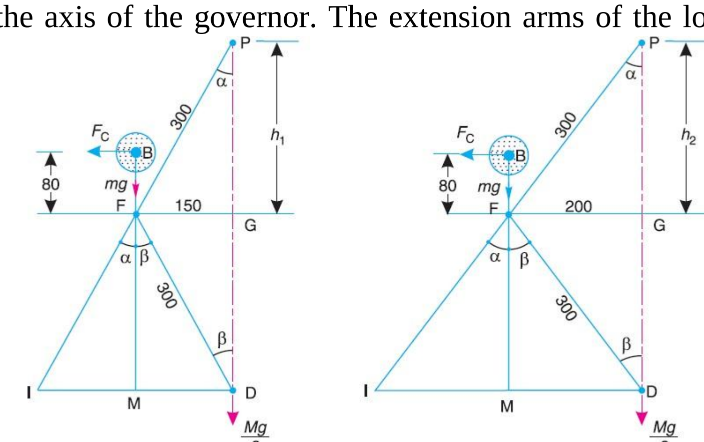
$$\cos\alpha = PK/PF = 200/250 = 0.8$$

$$T_1 \cos\alpha = mg + \frac{Mg}{2} = 3.2 \times 9.81 + \frac{25 \times 9.81}{2} = 154 \text{ N}$$

$$T_1 = \frac{154}{\cos\alpha} = \frac{154}{0.8} = 192.5 \text{ N} \quad \textbf{Ans.}$$

Problem – 4

A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and



All dimensions in mm.

(*a*) Minimum position. (*a*) Maximum position.

parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

$$h_2 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm} = 0.224 \text{ m}$$

$$FM = GD = PG = 224 \text{ mm} = 0.224 \text{ m}$$

$$\therefore \quad BM = BF + FM = 80 + 224 = 304 \text{ mm} = 0.304 \text{ m}$$

We know that $(N_2)^2 = \dfrac{FM}{BM}\left(\dfrac{m+M}{m}\right)\dfrac{895}{h_2}$ $\quad \ldots (\because \alpha = \beta \text{ or } q = 1)$

$$= \frac{0.224}{0.304}\left(\frac{10+100}{10}\right)\frac{895}{0.224} = 32\,385 \quad \text{or} \quad N_2 = 180 \text{ r.p.m.}$$

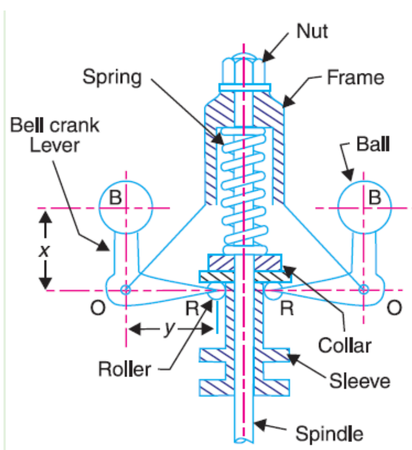
We know that range of speed

$$= N_2 - N_1 = 180 - 170 = 10 \text{ r.p.m. } \textbf{Ans.}$$

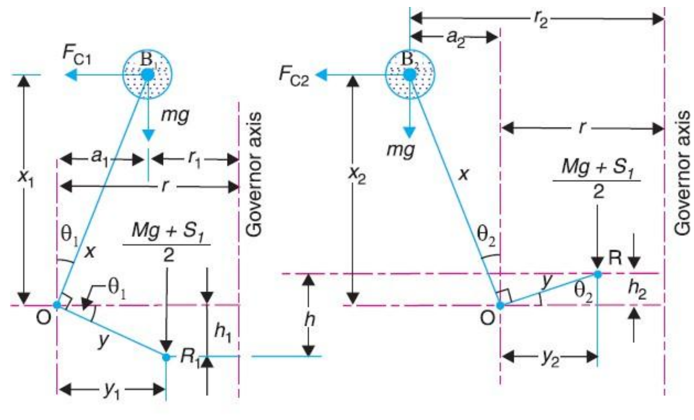
Lecture-5

Hartnell Governor

A Hartnell governor is a spring loaded governor as shown in Fig. 18.18. It consists of two bell crank levers pivoted at the points O,O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.



Hartnell governor



(*a*) Minimum position. (*b*) Maximum position.

Positions of Hartnell governor

Minimum position:

$$M \cdot g + S_1 = 2F_{C1} \times \frac{x}{y}$$

Maximum Position:

$$M \cdot g + S_2 = 2F_{C2} \times \frac{x}{y}$$

Problem-5:

In a spring loaded Hartnell type governor, the extreme radii of rotation of the balls are 80 mm and 120 mm. The ball arm and the sleeve arm of the bell crank lever are equal in length. The mass of each ball is 2 kg. If the speeds at the two extreme positions are 400 and 420 r.p.m., find : 1. the initial compression of the central spring, and 2. the spring constant.

Given:

r_1 = 80 mm = 0.08 m ;r_2 = 120 mm = 0.12 m ; $x = y$; m = 2 kg ; N_1 = 400r.p.m. or ξ_1 = 2 π × 400/60 = 41.9 rad/s ; N_2 = 420 r.p.m. or ξ_2= 2 π × 420/60 = 44 rad/s

Initial compression of the central spring

$$F_{C1} = m\,(\omega_1)^2\, r_1 = 2\,(41.9)^2\ \ 0.08 = 281 \text{ N}$$

$$F_{C2} = m\,(\omega_2)^2\, r_2 = 2\,(44)^2\ \ 0.12 = 465 \text{ N}$$

For Minimum Position:

$$M \cdot g + S_1 = 2\,F_{C1} \times \frac{x}{y}$$

$$S_1 = 2\,F_{C1} = 2 \times 281 = 562 \text{ N}$$

For Maximum Position

$$M \cdot g + S_2 = 2\,F_{C2} \times \frac{x}{y}$$

$$S_2 = 2\,F_{C2} = 2 \times 465 = 930 \text{ N}$$

Lift of the sleeve

$$h = (r_2 - r_1)\,\frac{y}{x} = r_2 - r_1 = 120 - 80 = 40 \text{ mm}$$

Stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{930 - 562}{40} = 9.2 \text{ N/mm}$$

initial compression of the central spring

$$= \frac{S_1}{s} = \frac{562}{9.2} = 61 \text{ mm} \quad \textbf{Ans.}$$

Problem-6:

In a spring controlled governorof the type, as shown in Fig, the mass of each ball is1.5 kg and the mass of the sleeve is 8 kg. The two arms ofthe bell crank lever are at right angles and their lengthsare OB = 100 mm and OA = 40 mm. The distance ofthe fulcrum O of each bell crank lever from the axis ofrotation is 50 mm and minimum radius of rotation of thegovernor balls is also 50 mm. The correspondingequilibrium speed is 240 r.p.m. and the sleeve is requiredto lift 10 mm for an increase in speed of 5 per cent. Findthe stiffness and initial compression of the spring.

Given:

m = 1.5 kg; M = 8 kg; $OB = x$= 100 mm = 0.1 m; $OA = y$ = 40 mm = 0.04 m; r = 50 mm = 0.05 m; r_1 = 50 mm = 0.05 m; N_1 = 240 r.p.m. orξ_1= 2π × 240/60 = 25.14 rad/s; h = 10 mm = 0.01 m; Increase in speed = 5%

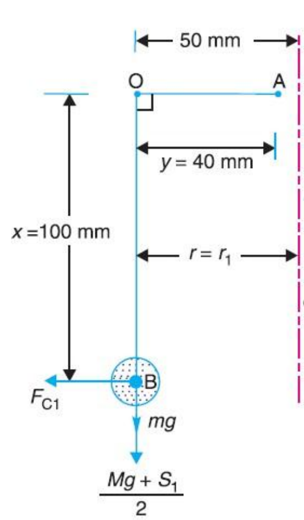
The spring controlled governor of the type, as shown in Fig, has the pivots for the bell crank lever on the moving sleeve. The spring is compressed between the sleeve and the cap which is fixed to the end of the governor shaft. The simplest way of analysing this type of governor is by taking moments about the instantaneous centre of all the forces which act on one of the bell crank levers. The minimum position of the governor is shown in Fig(a).

$$F_{C1} = m\,(\omega_1)^2\,r_1 = 1.5\,(25.14)^2\,0.05 = 47.4 \text{ N}$$

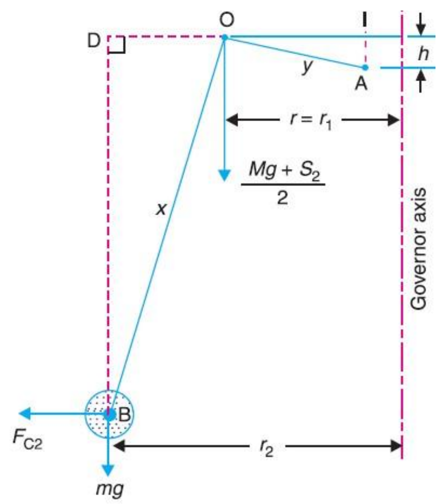
$$F_{C1} \times x = \left(m \cdot g + \frac{M \cdot g + S_1}{2}\right) OA$$

$$47.4 \times 0.1 = \left(1.5 \times 9.81 + \frac{8 \times 9.81 + S_1}{2}\right) 0.04 = 0.6 + 1.57 + 0.02\,S_1$$

$$4.74 = 2.17 + 0.02\,S_1 \quad \text{or} \quad S_1 = \frac{4.74 - 2.17}{0.02} = 128.5 \text{ N}$$



(b) Minimum position.



(b) Maximum position.

The maximum position of the governor is shown in Fig.(b). From the geometry of thefigure,

$$\frac{r_2 - r_1}{x} = \frac{h}{y} \quad \text{or} \quad r_2 = r_1 + h \times \frac{x}{y} = 0.05 + 0.01 \times \frac{0.1}{0.04} = 0.075 \text{ m}$$

Since the increase in speed is 5%, therefore the maximum equilibrium speed of rotation,

$$N_2 = N_1 + 0.05\, N_1 = 1.05\, N_1 = 1.05 \times 240 = 252 \text{ r.p.m.}$$

$$\omega_2 = 2\,\pi \times 252/60 = 26.4 \text{ rad/s}$$

∴ Centrifugal force acting on the ball at the maximum equilibrium speed,

$$F_{C2} = m\,(\omega_2)^2\, r_2 = 1.5\,(26.4)^2\, 0.075 = 78.4 \text{ N}$$

Let S_2 = Spring force at the maximum equilibrium speed.

The instantaneous centre in this case lies at I as shown in Fig. 18.25 (b). From the geometry igure,

$$OI = \sqrt{(OA)^2 - (IA)^2} = \sqrt{y^2 - h^2} = \sqrt{(0.04)^2 - (0.01)^2} = 0.0387 \text{ m}$$

$$BD = \sqrt{(OB)^2 - (OD)^2} = \sqrt{x^2 - (r_2 - r_1)^2}$$

$$= \sqrt{(0.1)^2 - (0.075 - 0.05)^2} = 0.097 \text{ m}$$

$$ID = OI + OD = 0.0387 + (0.075 - 0.05) = 0.0637 \text{ m}$$

Now taking moments about I,

$$F_{C2} \times BD = m.g \times ID + \frac{M.g + S_2}{2} \times OI$$

$$78.4 \times 0.097 = 1.5 \times 9.81 \times 0.0637 + \left(\frac{8 \times 9.81 + S_2}{2}\right) 0.0387$$

$$7.6 = 0.937 + 1.52 + 0.02\, S_2 = 2.457 + 0.019\, S_2$$

$$\therefore \qquad S_2 = \frac{7.6 - 2.457}{0.019} = 270.7 \text{ N}$$

We know that stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{270.7 - 128.5}{10} = 14.22 \text{ N/mm} \quad \textbf{Ans.}$$

Initial compression of the spring

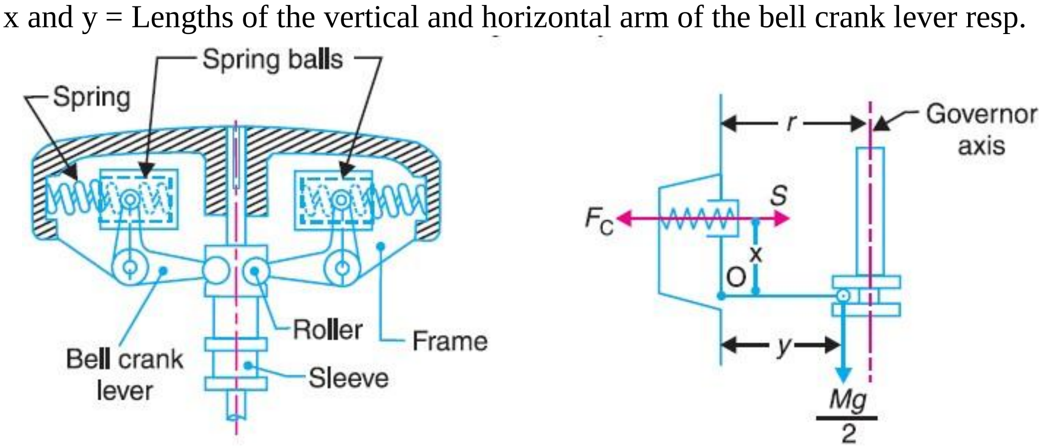
We know that initial compression of the spring

$$= \frac{S_1}{s} = \frac{128.5}{14.22} = 9.04 \text{ mm} \quad \textbf{Ans.}$$

Hartung Governor

A spring controlled governor of the Hartung type is shown in Fig (a). In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.

S = spring force,
FC = Centrifugal force,
M = Mass on the sleeve, and
x and y = Lengths of the vertical and horizontal arm of the bell crank lever resp.



(a) (b)

$$F_C \times x = S \times x + \frac{M.g}{2} \times y$$

Lecture-
7

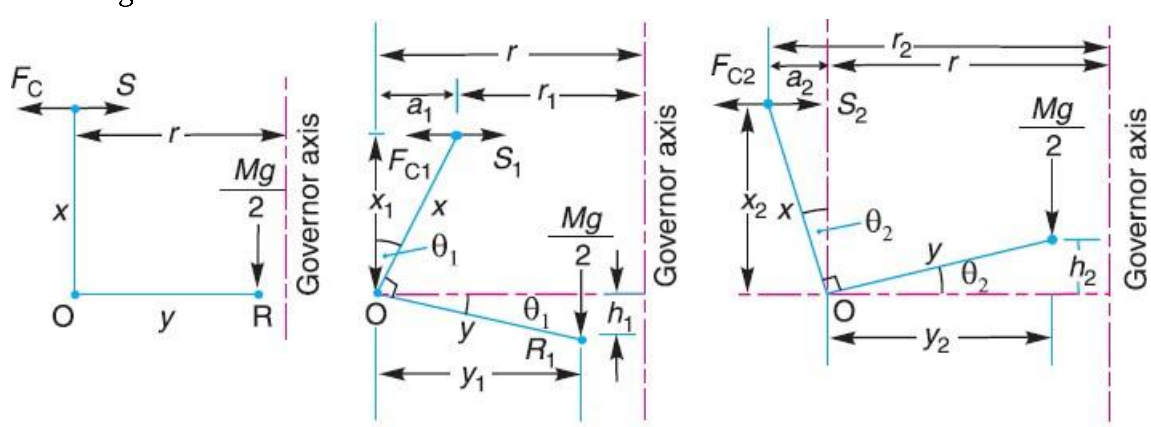
Problem – 6

In a spring-controlled governor of the Hartung type, the length of the ball and sleeve arms are 80 mm and 120 mm respectively. The total travel of the sleeve is 25 mm. In the mid position, each spring is compressed by 50 mm and the radius of rotation of the mass centres is 140 mm. Each ball has a mass of 4 kg and the spring has a stiffness of 10 kN/m of compression. The equivalent mass of the governor gear at the sleeve is 16 kg. Neglecting the moment due to the revolving masses when the arms are inclined, determine the ratio of the range of speed to the mean speed of the governor. Find, also, the speed in the mid-position.

Given:

x = 80 mm = 0.08 mm; y = 120 mm = 0.12 m ; h = 25 mm = 0.025 m ; r = 140 mm = 0.14 m ; m = 4 kg ; s = 10 kN/m = 10×10^3 N/m ; M = 16 kg ; Initial compression = 50 mm = 0.05 m

Mean speed of the governor



(*a*) Mean position (*b*) Minimum position (*c*) Maximum position

Let ω = Mean angular speed in rad/s, and

N = Mean speed in r.p.m.

We know that the centrifugal force acting on the ball spring,

$$F_C = m.\omega^2.r = 4 \times \omega^2 \times 0.14 = 0.56\ \omega^2 \text{ N}$$

and Spring force, S = Stiffness × Initial compression = $10 \times 10^3 \times 0.05 = 500$ N

Now taking moments about point O, neglecting the moment due to the revolving masses, we have

$$F_C \times x = S \times x + \frac{M.g}{2} \times y$$

$$0.56\ \omega^2 \times 0.08 = 500 \times 0.08 + \frac{16 \times 9.81}{2} \times 0.12 = 40 + 9.42 = 49.42$$

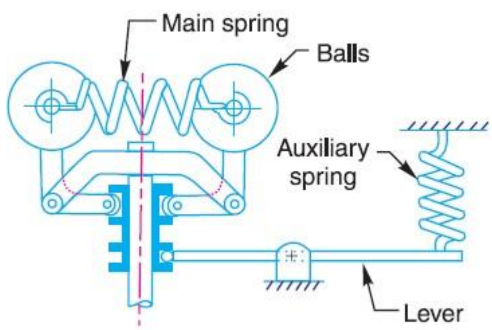
$$\therefore \quad \omega^2 = \frac{49.42}{0.56 \times 0.08} = 1103 \quad \text{or} \quad \omega = 33.2 \text{ rad/s}$$

and

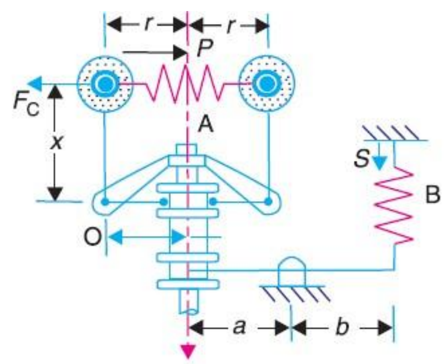
$$N = \frac{33.23 \times 60}{2\pi} = 317 \text{ r.p.m.} \quad \textbf{Ans.}$$

Wilson-Hartnell Governor

A Wilson-Hartnell governor is a governor in which the balls are connected by a spring in tension as shown in Fig. An auxiliary spring is attached to the sleeve mechanism through a lever by means of which the equilibrium speed for a given radius may be adjusted. The main spring may be considered of two equal parts each belonging to both the balls. The line diagram of a Wilson- Hartnell governor is shown in Fig.



Wilson-Hartnell governor.



Line diagram of Wilson-Hartnell governor.

$$4\,s_b + \frac{s_a}{2}\left(\frac{y}{x}\times\frac{b}{a}\right)^2 = \frac{F_{C2}-F_{C1}}{r_2-r_1}$$

Sensitiveness of Governors

Sensitiveness is defined as the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

Stability of Governors

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e. there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

Note: A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

Isochronous Governors

A governor is said to be isochronous when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

$$M.g + S_1 = 2\,F_{C1} \times \frac{x}{y} = 2 \times m \left(\frac{2\pi N_1}{60}\right)^2 r_1 \times \frac{x}{y}$$

$$M.g + S_2 = 2\,F_{C2} \times \frac{x}{y} = 2 \times m \left(\frac{2\pi N_2}{60}\right)^2 r_2 \times \frac{x}{y}$$

Haunting

A governor is said to be hunt if the speed of the engine fluctuates continuously above andbelow the mean speed. This is caused by a too sensitive governor which changes the fuel supplyby a large amount when a small change in the speed of rotation takes place.

Effort and Power of a Governor

The effort of a governor is the mean force exerted at the sleeve for a given percentage change of speed* (or lift of the sleeve). It may be noted that when the governor is running steadily, there is no force at the sleeve. But, when the speed changes, there is a resistance at the sleeve which opposes its motion. It is assumed that this resistance which is equal to the effort, varies uniformly from a maximum value to zero while the governor moves into its new position of equilibrium.

The power of a governor is the work done at the sleeve for a given percentage change of speed. It is the product of the mean value of the effort and the distance through which the sleeve moves.

Power = Mean effort × lift of sleeve

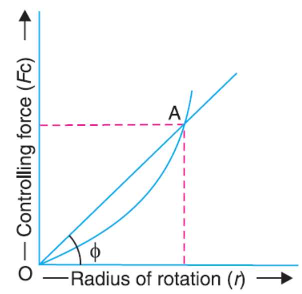
Lecture-8

Controlling force:

Governor running at a steady speed, the inward force acting on the rotating balls is known as controlling force. It is equal and opposite to the centrifugal reaction. The Controlling force is given by

Controlling force, FC = $m.\xi^2.r$

The controlling force is provided by the weight of the sleeve and balls as in Porter governor and by the spring and weight as in Hartnell governor (or spring controlled governor). When the graph between the controlling force (F C) as ordinate and radius of rotation of the balls (r) as abscissa is drawn, then the graph obtained is known as controlling force diagram. This diagram enables the stability and sensitiveness of the governor to be examined and also shows clearly the effect of friction.



Controlling Force Diagram for Porter Governor

$$F_C = m.\omega^2.r = m\left(\frac{2\,\pi\,N}{60}\right)^2 r$$

$$N^2 = \frac{1}{m}\left(\frac{60}{2\pi}\right)^2\left(\frac{F_C}{r}\right) = \frac{1}{m}\left(\frac{60}{2\pi}\right)^2(\tan\phi)$$

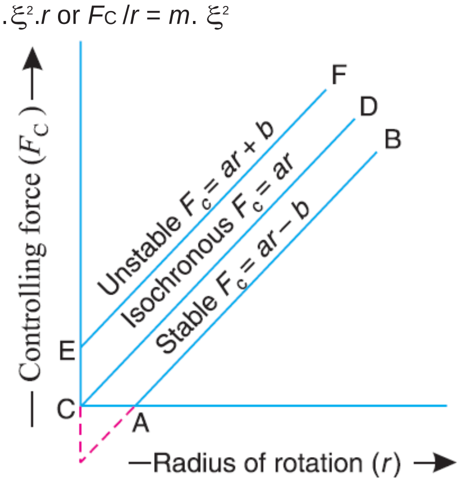
$$N = \frac{60}{2\pi}\left(\frac{\tan\phi}{m}\right)^{1/2}$$

where Φis the angle between the axis of radius of rotation and a line joining a given point (say A) onthe curve to the origin O.

<u>Controlling Force Diagram for Spring-controlled Governors</u>

The controlling force diagram for the spring controlled governors is a straight line, as shownin Fig. We know that controlling force,

$F_C = m.\xi^2.r$ or $F_C/r = m.\ \xi^2$



For the governor to be stable, the controlling force (F_C) must increase as the radius of rotation(r) increases, i.e. F_C / r must increase as r increases. Hence the controlling force line AB when produced must intersect the controlling force axis below the origin, as shown in Fig. The relation between the controlling force (F_C)

and the radius of rotation (r) for the stability of spring controlled governors is given by the following equation

$F_C = a.r - b$--(i)

Where a and b are constants.

The value of b in equation (i) may be made either zero or positive by increasing the initial tension of the spring. If b is zero, the controlling force line CD passes through the origin and the governor becomes isochronous because F_C /r will remain constant for all radii of rotation. The relation between the controlling force and the radius of rotation, for an isochronous governor is, therefore,

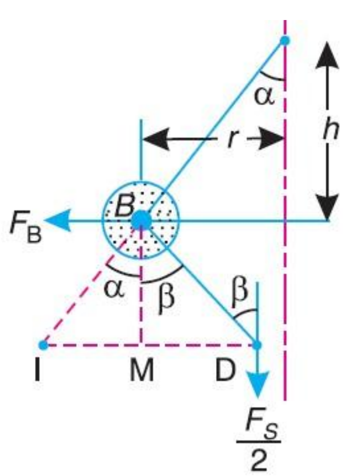
$F_C = a.r$--(ii)

If b is greater than zero or positive, then F_C /r decreases as r increases, so that the equilibrium speed of the governor decreases with an increase of the radius of rotation of balls, which is impracticable. Such a governor is said to be unstable and the relation between the controlling force and the radius of rotation is, therefore

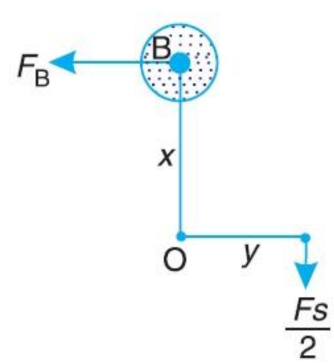
$F_C = a.r + b$--(iii)

<u>Coefficient of Insensitiveness</u>

We have assumed the governor to be frictionless. In actual practice, there is always friction in the joints and operating mechanism of the governor. Since the frictional force always acts in the opposite direction to that of motion, therefore, when the speed of rotation decreases, the friction prevents the downward movement of the sleeve and the radial inward movement of the balls. On the other hand, when the speed of rotation increases, the friction prevents the upward movement of the sleeve and radial outward movement of the balls.



(a) Porter governor.



(b) Spring loaded governor.

Lecture-9

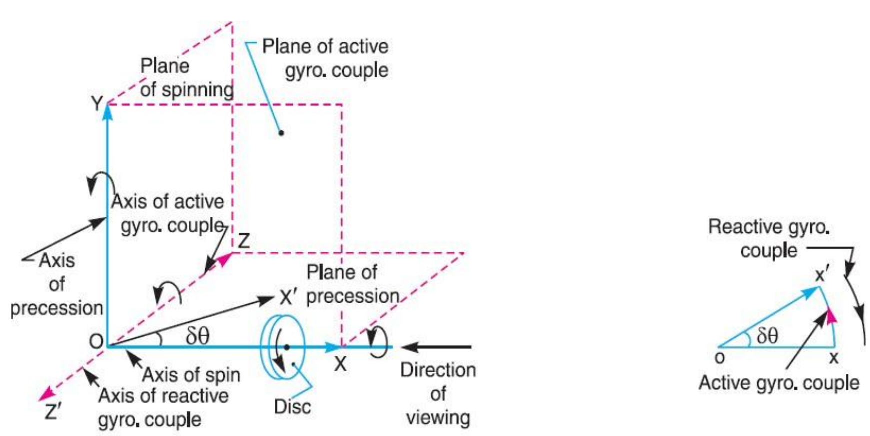
Gyroscopic Couple:

Consider a disc spinning with an angular velocity ξ.rad/s about the axis of spin OX, in anticlockwise direction when seen from the front, as shown in Fig(a). Since the plane in which the disc is rotating is parallel to the plane YOZ, therefore it is called plane of spinning. The planeXOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane aboutan axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY. Inother words, the axis of spin is said to be rotating or processing about an axis OY (which is perpendicular to both the axes OX and OZ) at an angular velocity ξ_P rap/s. This horizontal plane XOZ is called plane of precession and OY is the axis of precession.

I = Mass moment of inertia of the disc about *OX,* andξ = Angular velocity of the disc.

Angular momentum of the disc= *I*.ξ

Since the angular momentum is a vector quantity, therefore it may be represented by the vector <u>*ox*</u>., as shown in Fig.(*b*). The axis of spin *OX* is also rotating anticlockwise when seen from the top about the axis *OY*. Let the axis *OX* is turned in the plane *XOZ* through a small angle δ0.radians to the position *OX'*., in time δ*t*seconds. Assuming the angular velocity ξ.to be constant, the angular momentum will now be represented by vector *OX'*.



(*a*) (*b*)

Change in angular momentum = I.ξ.δ0

Rate of change of angular momentum

$$= I.\omega \times \frac{\delta\theta}{dt}$$

The rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession

$$C = I.\omega.\omega_P$$

Problem – 7

A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis?

Given: d = 300 mm or r = 150 mm = 0.15 m ;m = 5 kg ; l = 600 mm = 0.6 m ; N = 300 r.p.m. or $\xi = 2\pi \times 300/60$ = 31.42 rad/s.

$I = m.r^2/2 = 5(0.15)^2/2 = 0.056$ kg-m^2

$C = m.g.l = 5 \times 9.81 \times 0.6 = 29.43$ N-m

ω_P = Speed of precession.

We know that couple (C),

$$29.43 = I.\omega.\omega_P = 0.056 \times 31.42 \times \omega_P = 1.76\ \omega_P$$

$\therefore \quad \omega_P = 29.43/1.76 = 16.7$ rad/s **Ans.**

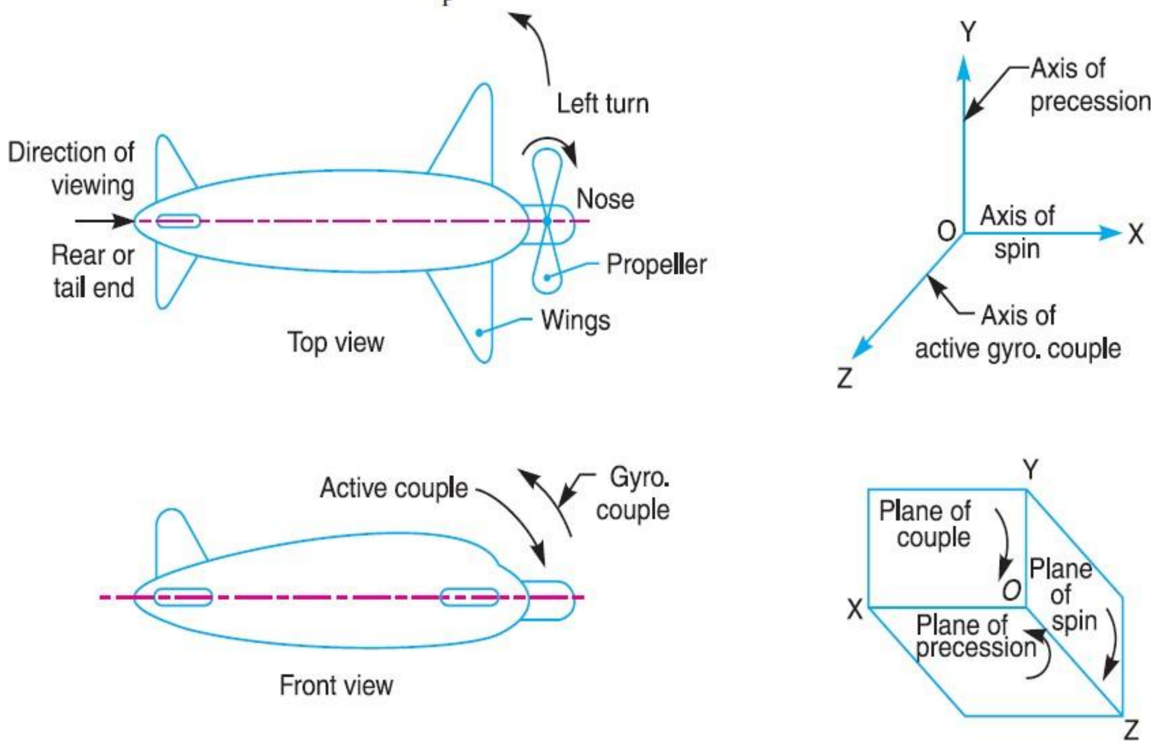
Lecture-10

Effect of the Gyroscopic Couple on an Aeroplane

The top and front views of an aeroplane are shown in Fig (a). Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left.

∴ Gyroscopic couple acting on the aeroplane,

$$C = I.\omega.\omega_P$$



Aeroplane taking left turn

Notes: 1. when the aeroplane takes a right turn under similar conditions as discussed above, the effect of the reactive gyroscopic couple will be to dip the nose and raise the tail of the aeroplane.
2. When the engine or propeller rotates in anticlockwise direction when viewed from the rear or tail end and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to dip the nose and raise the tail of the aeroplane.
3. When the aeroplane takes a right turn under similar conditions as mentioned in note 2 above, theeffect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aeroplane.
4. When the engine or propeller rotates in clockwise direction when viewed from the front and theaeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to raise the tail and dip the noseof the aeroplane.
5. When the aeroplane takes a right turn under similar conditions as mentioned in note 4-above, theeffect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aeroplane.

Problem – 8

An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

Given : R = 50 m ; v = 200 km/hr = 55.6 m/s ; m = 400 kg ; k = 0.3 m ;

N = 2400 r.p.m. or $\xi = 2\pi \times 2400/60 = 251$ rad/s

We know that mass moment of inertia of the engine and the propeller,

$I = m.k^2 = 400(0.3)^2 = 36$ kg-m^2

and angular velocity of precession,

$\xi_P = v/R = 55.6/50 = 1.11$ rad/s

We know that gyroscopic couple acting on the aircraft,

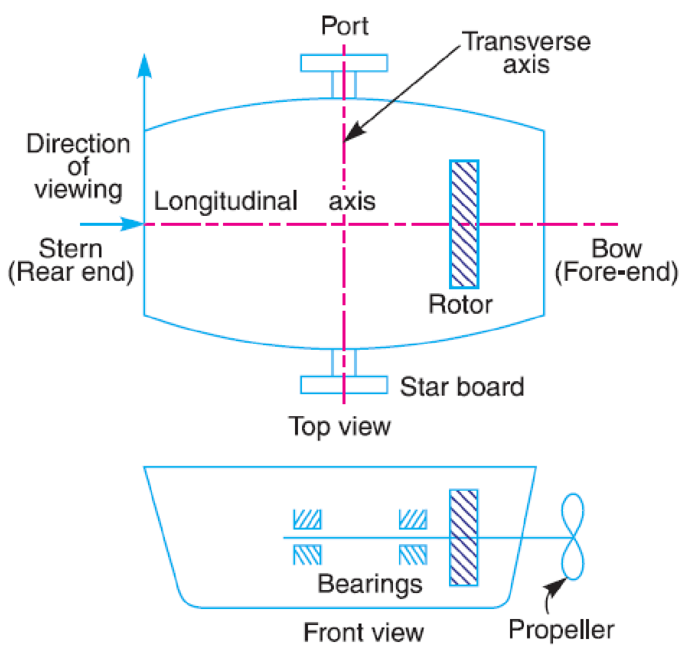
$C = I.\xi.\xi_P = 36 \times 251.4 \times 1.11 = 100\ 46$ N-m = **10.046 kN-m**

When the aeroplane turns towards left, the effect of thegyroscopic couple is to lift the nose upwards and tail downwards.

Terms Used in a Naval Ship

The top and front views of a naval ship are shown in Fig. The fore end of the ship iscalled bow and the rear end is known as stern or aft. The left hand and right hand sides of the ship,when viewed from the stern are called port and star-board respectively. We shall now discuss theeffect of gyroscopic couple on the naval ship in the following three cases:

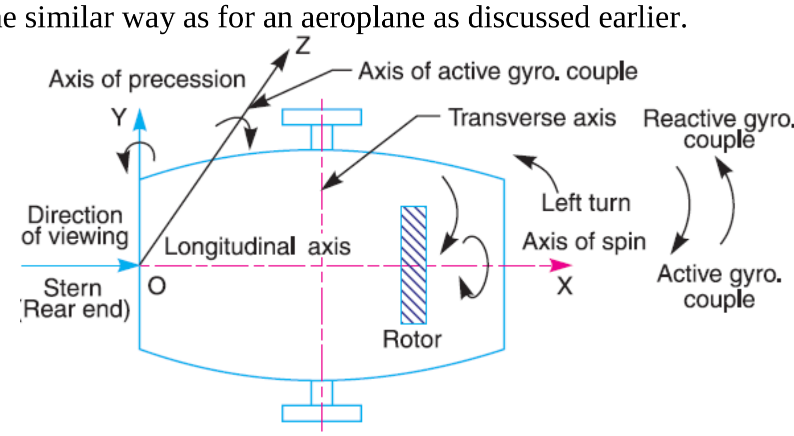
1. Steering, 2. Pitching and 3. Rolling.



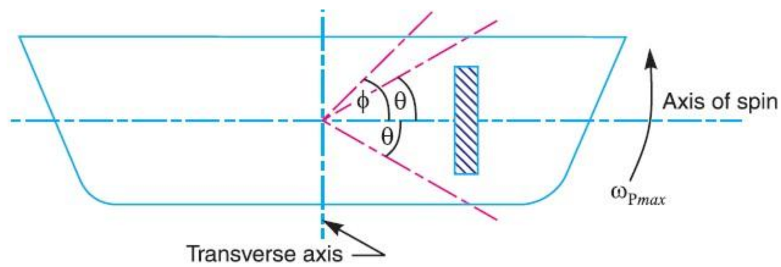
Lecture-11

Effect of Gyroscopic Couple on a Naval Ship during Steering

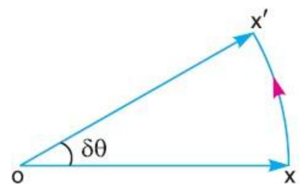
Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane as discussed earlier.



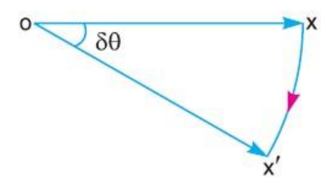
Effect of Gyroscopic Couple on a Naval Ship during Pitching



(*a*) Pitching of a naval ship



(*b*) Pitching upward



(*c*) Pitching downward

Effect of Gyroscopic Couple on a Naval Ship during Rolling

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship. In case of rolling of a ship, the axis of precession (i.e. longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Problem – 9

The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.

Given:

m = 8 t = 8000 kg; k = 0.6 m; N = 1800 r.p.m. or $\xi = 2\pi \times 1800/60 = 188.5$ rad/s; v = 100 km/h = 27.8 m/s; R = 75 m

Mass moment of inertia of the rotor,

$I = m.k^2 = 8000\,(0.6)^2 = 2880$ kg-m^2

Angular velocity of precession,

$\xi_P = v / R = 27.8 / 75 = 0.37$ rad/s

Gyroscopic couple,

$C = I.\xi.\xi_P = 2880 \times 188.5 \times 0.37 =$ **200 866 N-m**

When the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

Problem – 10

The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:

1. When the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.

2. When the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

Given: m = 3500 kg; k = 0.45 m; N = 3000 r.p.m. or $\xi = 2\pi \times 3000/60 = 314.2$ rad/s

1. *When the ship is steering to the left*

Given: R =100 m ; v = km/h = 10 m/s

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 3500\,(0.45)^2 = 708.75 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_p = v/R = 10/100 = 0.1 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\omega_p = 708.75 \times 314.2 \times 0.1 = 22\ 270 \text{ N-m}$$

$$= 22.27 \text{ kN-m} \textbf{ Ans.}$$

2. *When the ship is pitching with the bow falling*

Given: $t_p = 40$ s

Since the total angular displacement between the two extreme positions of pitching is 12° (*i.e.* $2\phi = 12°$), therefore amplitude of swing,

$$\phi = 12 / 2 = 6° = 6 \times \pi/180 = 0.105 \text{ rad}$$

and angular velocity of the simple harmonic motion,

$$\omega_1 = 2\pi / t_p = 2\pi / 40 = 0.157 \text{ rad/s}$$

We know that maximum angular velocity of precession,

$$\omega_p = \phi.\omega_1 = 0.105 \times 0.157 = 0.0165 \text{ rad/s}$$

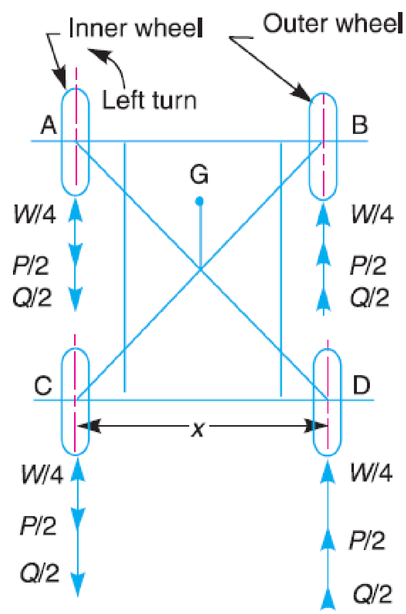
∴ Gyroscopic couple,

$$C = I.\omega.\omega_p = 708.75 \times 314.2 \times 0.0165 = 3675 \text{ N-m}$$

$$= 3.675 \text{ kN-m} \textbf{ Ans.}$$

Lecture-12

Stability of a Four Wheel Drive Moving in a Curved Path



1. *Effect of the gyroscopic couple*

Since the vehicle takes a turn towards left due to the precession and other rotating parts, therefore a gyroscopic couple will act.

We know that velocity of precession,

$$\omega_P = v/R$$

∴ Gyroscopic couple due to 4 wheels,

$$C_W = 4\, I_W.\omega_W.\omega_P$$

and gyroscopic couple due to the rotating parts of the engine,

$$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P \qquad \ldots (\because G = \omega_E/\omega_W)$$

∴ Net gyroscopic couple,

$$C = C_W \pm C_E = 4\, I_W.\omega_W.\omega_P \pm I_E.G.\omega_W.\omega_P$$
$$= \omega_W.\omega_P\,(4\, I_W \pm G.I_E)$$

The ***positive*** sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolves in opposite direction, then ***negative*** sign is used.

Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be P newtons. Then

$$P \times x = C \quad \text{or} \quad P = C/x$$

∴ Vertical reaction at each of the outer or inner wheels,

$$P/2 = C/2x$$

Note: We have discussed above that when rotating parts of the engine rotate in opposite directions, then –ve sign is used, *i.e.* net gyroscopic couple,

$$C = C_W - C_E$$

When $C_E > C_W$, then C will be –ve. Thus the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels.

2. *Effect of the centrifugal couple*

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle. We know that centrifugal force,

$$F_C = \frac{m \times v^2}{R}$$

∴ The couple tending to overturn the vehicle or overturning couple,

$$C_O = F_C \times h = \frac{m.v^2}{R} \times h$$

This overturning couple is balanced by vertical reactions, which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be Q. Then

$$Q \times x = C_O \quad \text{or} \quad Q = \frac{C_O}{x} = \frac{m.v^2.h}{R.x}$$

∴ Vertical reaction at each of the outer or inner wheels,

$$\frac{Q}{2} = \frac{m.v^2.h}{2R.x}$$

∴ Total vertical reaction at each of the outer wheel,

$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

and total vertical reaction at each of the inner wheel,

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

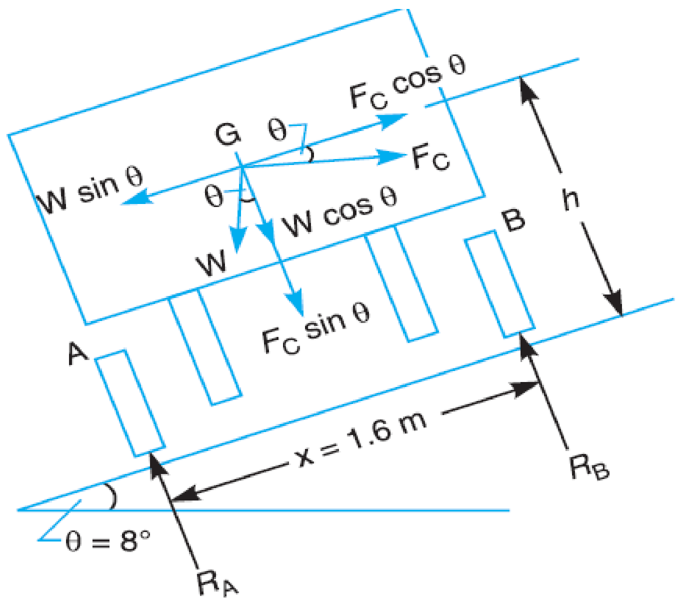
A little consideration will show that when the vehicle is running at high speeds, PI may be zero or even negative. This will cause the inner wheels to leave the ground thus tending to overturn the Automobile. In order to have the contact between the inner wheels and the ground, the sum of P/2 and Q/2 must be less than W/4.

Problem – 10:

A four-wheeled trolley car of total mass 2000 kg running on rails of 1.6 mgauge, rounds a curve of 30 m radius at 54 km/h. The track is banked at 8°. The wheels have an external diameter of 0.7 m and each pair with axle has a mass of 200 kg. The radius of gyration for each pair is 0.3 m. The height of centre of gravity of the car above the wheel base is 1 m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.

Given: m = 2000 kg ;x = 1.6 m ; R = 30 m ; v = 54 km / h = 15 m / s ; θ = 8° ; d_w= 0.7 m or r_w= 0.35 m ; m_1 = 200 kg ; k = 0.3 m ; h = 1 m

First of all, let us find the reactions R_A and R_B at the wheels A and B respectively. The various forces acting on the trolley car are shown in Fig.



Now taking moments about B,

$$R_A \times x = (W\cos\theta + F_C\sin\theta)\,\frac{x}{2} + W\sin\theta \times h - F_C\cos\theta \times h$$

$$\therefore \quad R_A = \left(m.g\cos\theta + \frac{m.v^2}{R}\sin\theta\right)\frac{1}{2} + \left(m.g\sin\theta - \frac{m.v^2}{R}\cos\theta\right)\frac{h}{x}$$

$$= \left(2000 \times 9.81\cos 8^\circ + \frac{2000\,(15)^2}{30}\sin 8^\circ\right)\frac{1}{2}$$

$$+ \left(2000 \times 9.81\sin 8^\circ - \frac{2000\,(15)^2}{30}\cos 8^\circ\right)\frac{1}{1.6}$$

$$= (19\,620 \times 0.9903 + 15\,000 \times 0.1392)\,\frac{1}{2}$$

$$+ (19\,620 \times 0.1392 - 15\,000 \times 0.9903)\,\frac{1}{1.6}$$

$$= (19\,430 + 2088)\,\frac{}{2} + (2731 - 14\,855)\,\frac{}{1.6}$$

$$= 10\,759 - 7577 = 3182 \text{ N}$$

$$\therefore \quad R_B = (R_A + R_B) - R_A = 21\,518 - 3182 = 18\,336 \text{ N}$$

We know that angular velocity of wheels,

$$\omega_W = \frac{v}{r_W} = \frac{15}{0.35} = 42.86 \text{ rad/s}$$

and angular velocity of precession,

$$\omega_P = \frac{v}{R} = \frac{15}{30} = 0.5 \text{ rad/s}$$

$\therefore$ Gyroscopic couple,

$$C = {}^{*}I\,\omega_W \cos\theta \times \omega_P = m_I.k^2.\omega_W \cos\theta.\omega_P \qquad ...(\because I = m_I.k^2)$$

$$= 200\,(0.3)^2\, 42.86 \cos 8^\circ \times 0.5 = 382 \text{ N-m}$$

Due to this gyroscopic couple, the car will tend to overturn about the outer wheels. Let P be the force at each pair of wheels or each rail due to the gyroscopic couple,

$$\therefore \quad P = C / x = 382 / 1.6 = 238.75 \text{ N}$$

We know that pressure (or total reaction) on the inner rail,

$$P_I = R_A - P = 3182 - 238.75 = 2943.25 \text{ N}$$ **Ans.**

and pressure on the outer rail,

$$P_O = R_B + P = 18\,336 + 238.75 = 18\,574.75 \text{ N}$$ **Ans.**