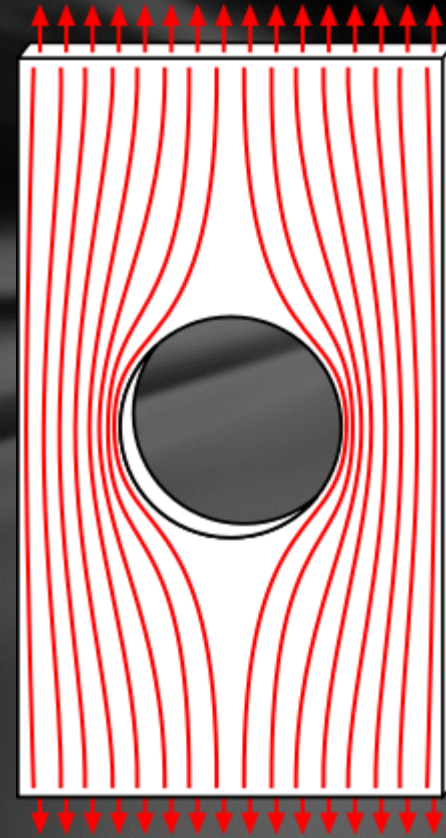


DESIGN OF MACHINE ELEMENTS

UNIT I STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS

Stress Concentration

- Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration***.
- A **stress concentration (stress raisers or stress risers)** is a location in an object where **stress** is concentrated. An object is strongest when force is evenly distributed over its area, so a reduction in area, e.g., caused by a crack, results in a localized increase in stress.
- A material can fail, via a **propagating crack**, when a concentrated stress exceeds the material's theoretical cohesive strength. The real **fracture** strength of a material is always lower than the theoretical value because most materials contain small cracks or contaminants (especially foreign particles) that concentrate stress.
- It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.



Internal **force lines** are denser near the hole

Theoretical or Form Stress Concentration Factor

- The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area.
- Mathematically, theoretical or form stress concentration factor,

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

- The value of K_t depends upon the material and geometry of the part.

Fatigue Stress Concentration Factor

- When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor.
- Mathematically, fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

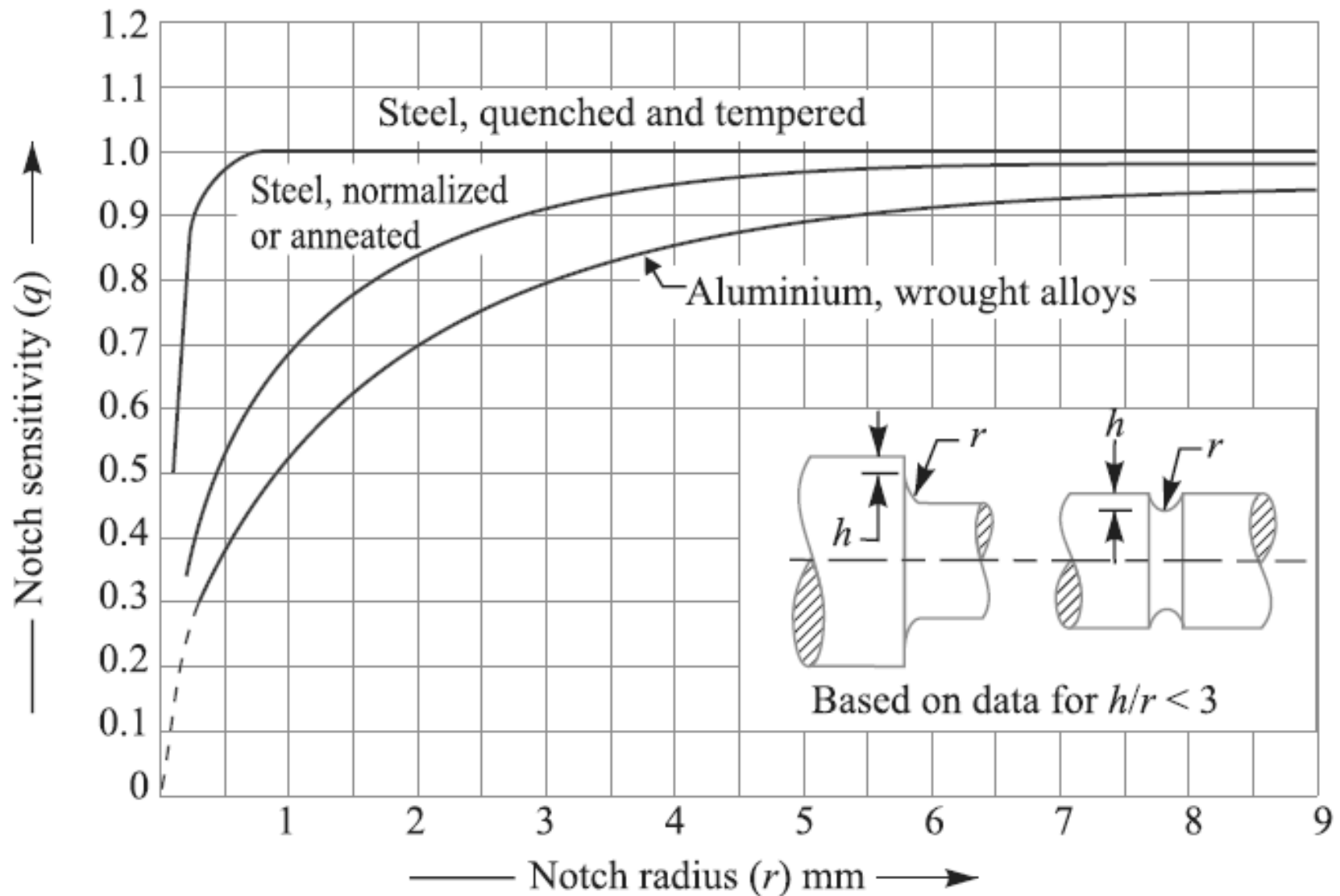
- **Notch Sensitivity:** It may be defined as the degree to which the theoretical effect of stress concentration is actually reached.
- **Notch Sensitivity Factor “q”:** Notch sensitivity factor is defined as the ratio of increase in the actual stress to the increase in the nominal stress near the discontinuity in the specimen.

$$q = \frac{K_f - 1}{K_t - 1}$$

Where, K_f and K_t are the fatigue stress concentration factor and theoretical stress concentration factor.

- The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material.

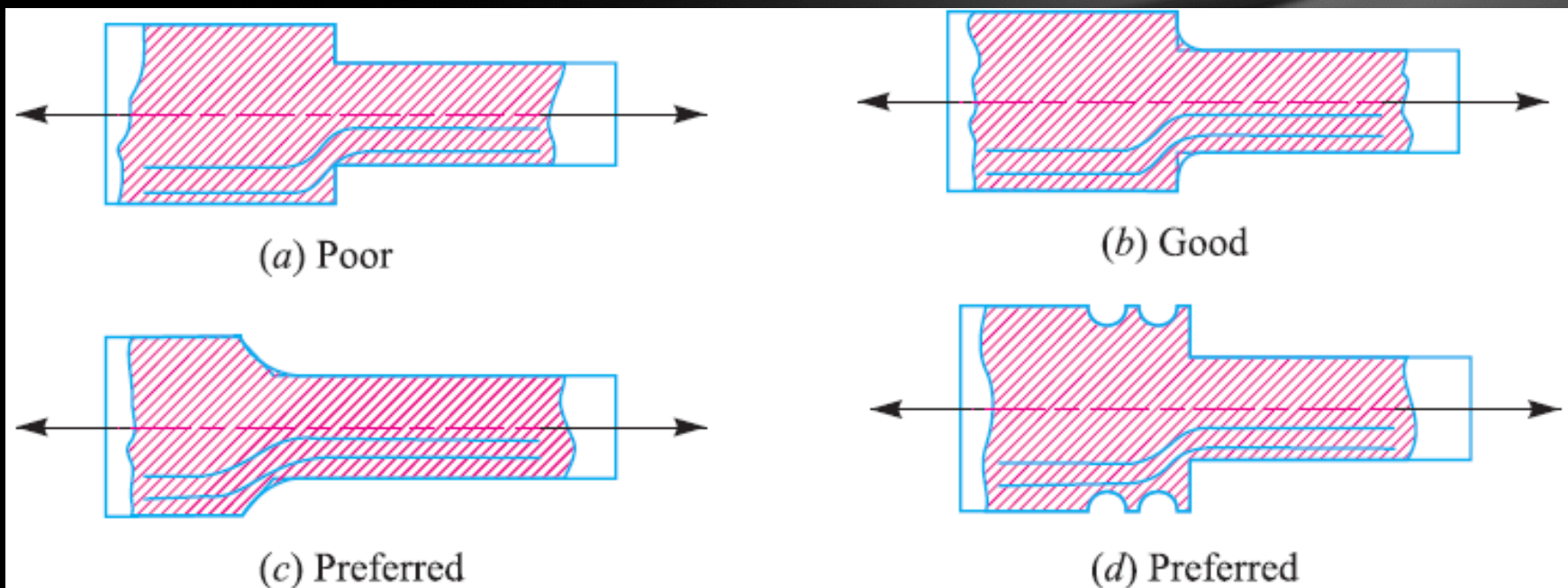
- Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in figure may be used for determining the values of q for two steels.



Methods to reduce stress concentration

- The presence of stress concentration can not be totally eliminated but it may be reduced to some extent.
- A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines.
- The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

- In Fig. (a), we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. (b) and (c) to give more equally spaced flow lines.
- It may be noted that it is not practicable to use large radius fillets as in case of ball and roller bearing mountings. In such cases, notches may be cut as shown in Fig. (d).



- Following figures show the several ways of reducing the stress concentration in shafts and other cylindrical members with shoulders, holes and threads :

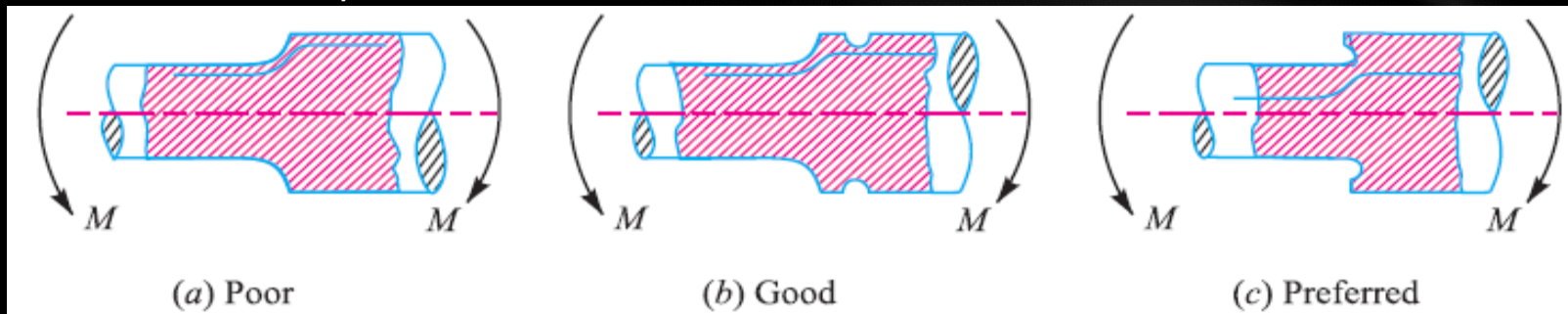


Fig. 6.9. Methods of reducing stress concentration in cylindrical members with shoulders.



Fig. 6.10. Methods of reducing stress concentration in cylindrical members with holes.

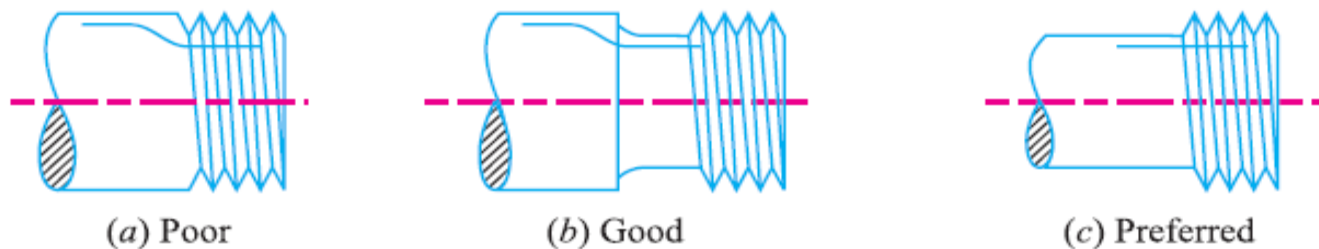


Fig. 6.11. Methods of reducing stress concentration in cylindrical members with holes.

- The stress concentration effects of a press fit may be reduced by making more gradual transition from the rigid to the more flexible shaft.

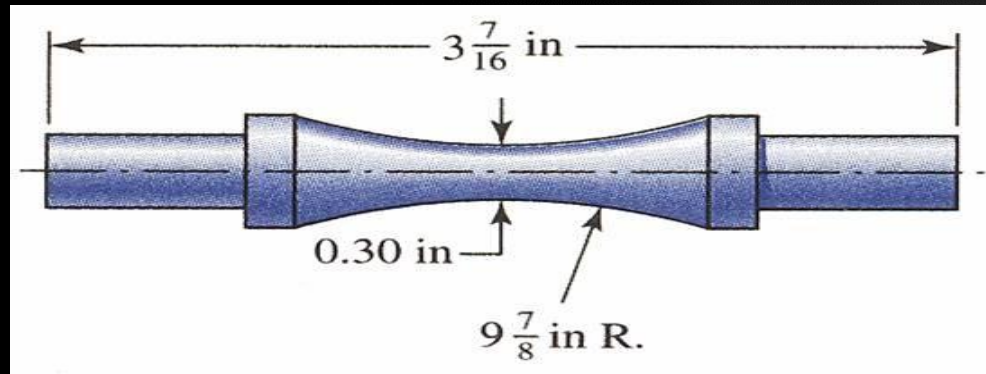
Factors to be Considered while Designing Machine Parts to Avoid Fatigue Failure

- The following factors should be considered while designing machine parts to avoid fatigue failure:
 1. The variation in the size of the component should be as gradual as possible.
 2. The holes, notches and other stress raisers should be avoided.
 3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.
 4. The parts should be protected from corrosive atmosphere.
 5. A smooth finish of outer surface of the component increases the fatigue life.
 6. The material with high fatigue strength should be selected.
 7. The residual compressive stresses over the parts surface increases its fatigue strength.

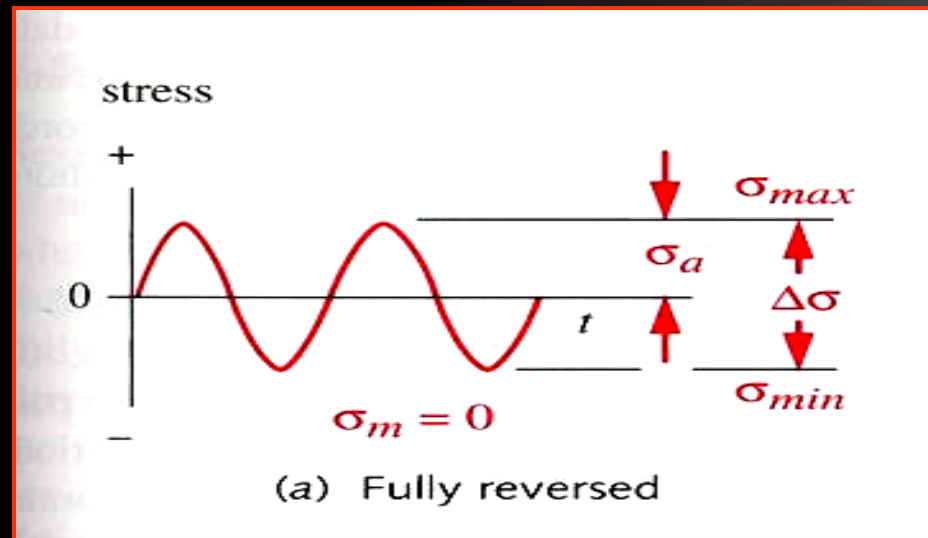
Endurance limit and Fatigue failure

- It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue**.
- The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication.
- The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

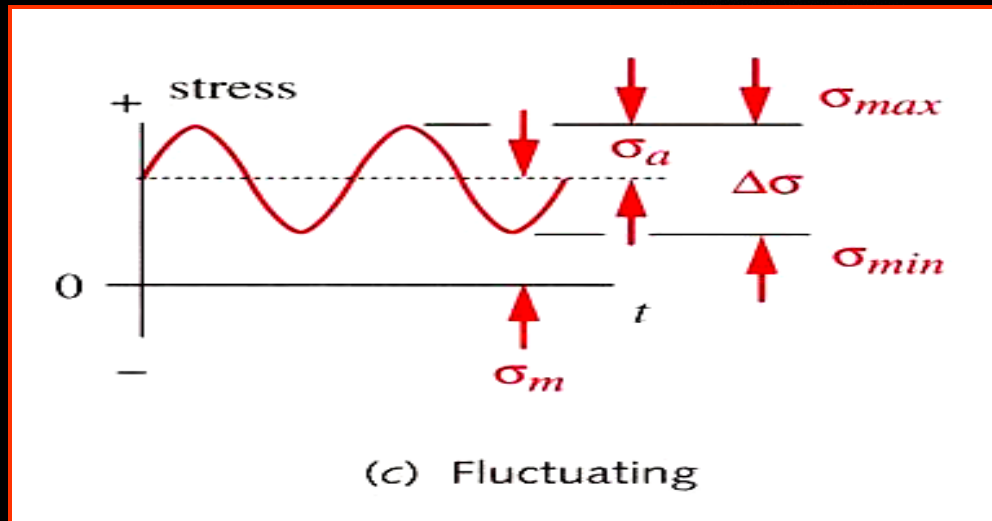
- A standard mirror polished specimen, as shown in figure is rotated in a fatigue testing machine while the specimen is loaded in bending.



- As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive.
- In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. (a).



- **Endurance** or **Fatigue limit** (σ_e) is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).
- It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term **endurance strength** may be used when referring the fatigue strength of the material.
- It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.
- We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig.(a). In actual practice, many machine members undergo different range of stress than the completely reversed stress.
- The stress **verses** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig. (c). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v .



➤ The following relations are derived from Fig. (c):

Alternating stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Mean stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Factors affecting endurance limit

1) SIZE EFFECT:

- The strength of large members is lower than that of small specimens.
- This may be due to two reasons.
 - The larger member will have a larger distribution of weak points than the smaller one and on an average, fails at a lower stress.
 - Larger members have larger surface Area. This is important because the imperfections that cause fatigue failure are usually at the surface.

❖ Effect of size:

Increasing the size (especially section thickness) results in larger surface area and creation of stresses.

This factor leads to increase in the probability of crack initiation.

This factor must be kept in mind while designing large sized components.

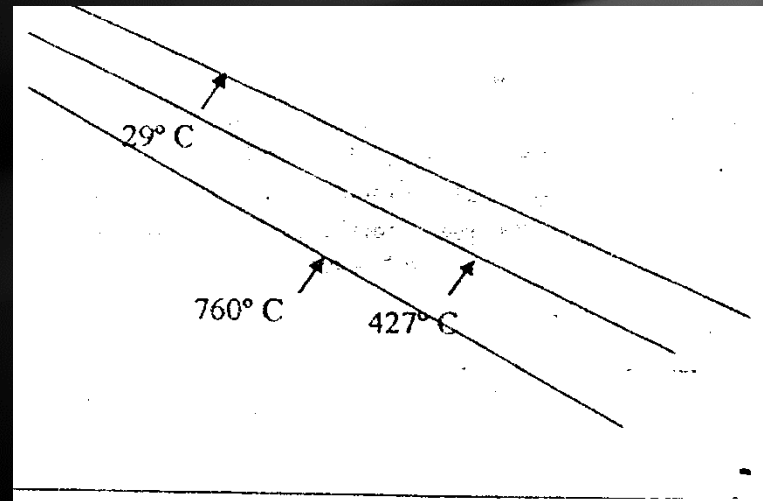
2) SURFACE ROUGHNESS:

- **almost all fatigue cracks nucleate at the surface of the members.**
- **The conditions of the surface roughness and surface oxidation or corrosion are very important.**
- **Experiments have shown that different surface finishes of the same material will show different fatigue strength.**
- **Methods which Improve the surface finish and those which introduce compressive stresses on the surface will improve the fatigue strength.**
- **Smoothly polished specimens have higher fatigue strength.**
- **Surface treatments. Fatigue cracks initiate at free surface, treatments can be significant**
- **Plating, thermal or mechanical means to induce residual stress**

3) EFFECT OF TEMPERATURE:

- Fatigue tests on metals carried out at below room temperature shows that fatigue strength increases with decreasing temperature.

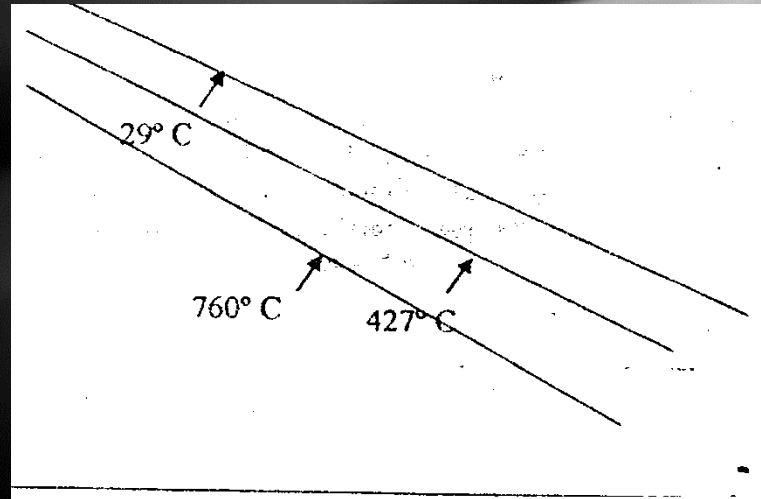
Stress amplitude



No. of cycles to Failure

Higher the temperature,
lower the fatigue strength.

Stress amplitude



No. of cycles to Failure

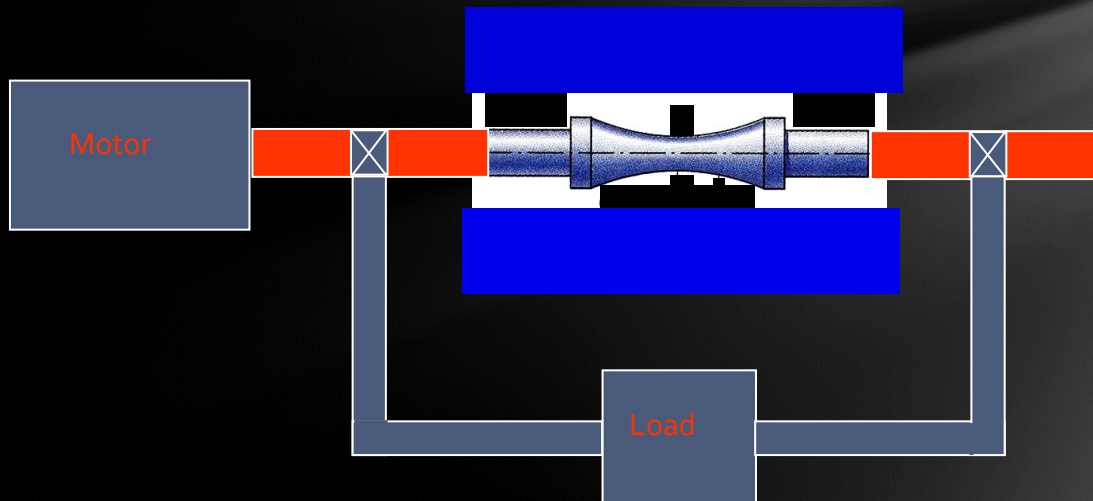
4) Effect of metallurgical variables;

- Fatigue strength generally increases with increase in UTS
- Fatigue strength of quenched & tempered steels (tempered martensitic structure) have better fatigue strength
- Finer grain size show better fatigue strength than coarser grain size.
- Non-metallic inclusions either at surface or sub-surface reduces' the fatigue strength

S-N Diagram

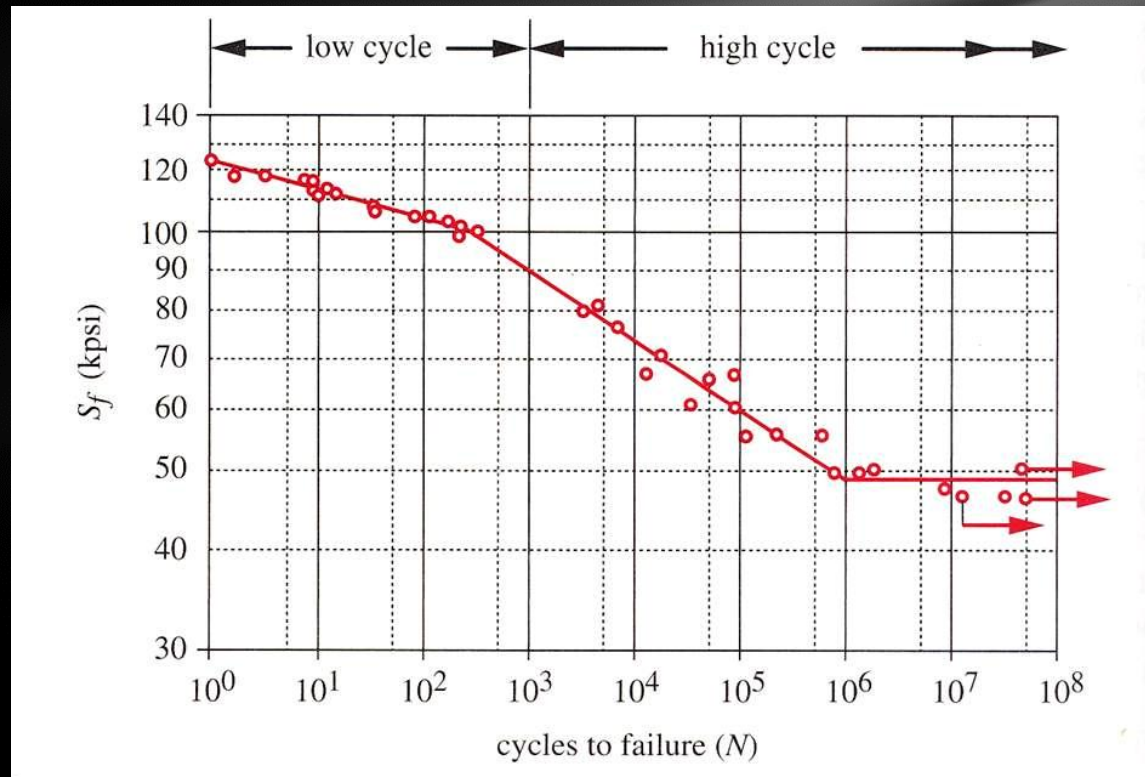
- Fatigue strength of material is determined by R.R. Moore rotating beam machine. The surface is polished in the axial direction. A constant bending load is applied.

Typical testing apparatus, pure bending

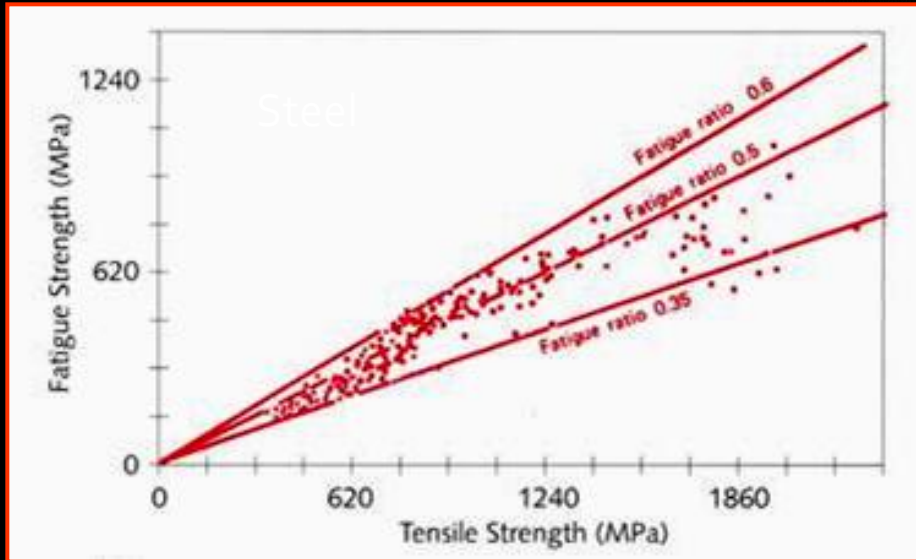


Rotating beam machine – applies fully reverse bending stress

- A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in figure.
- A little consideration will show that if the stress is kept below a certain value the material will not fail whatever may be the number of cycles.
- This stress, as represented by dotted line, is known as **endurance** or **fatigue limit** (σ_e).
- It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).



Relationship Between Endurance Limit and Ultimate Strength

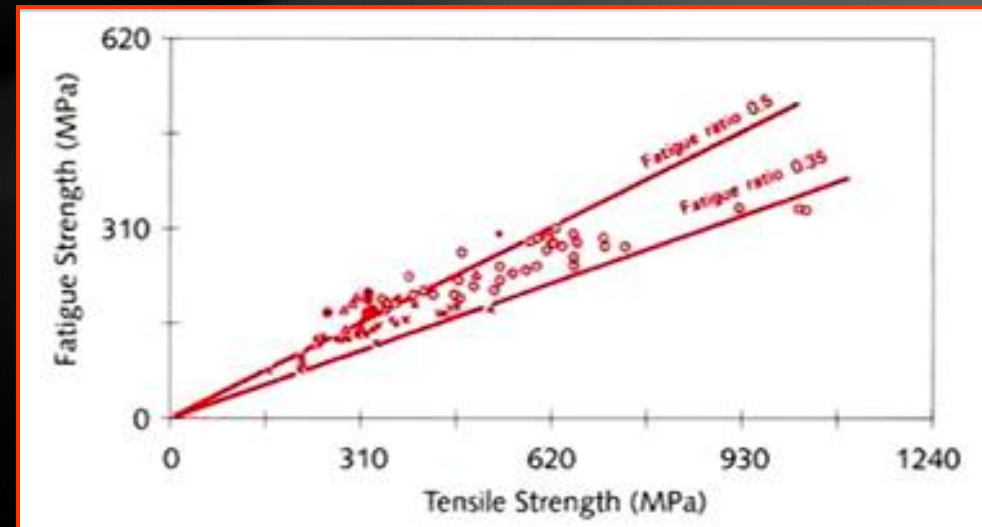


Steel

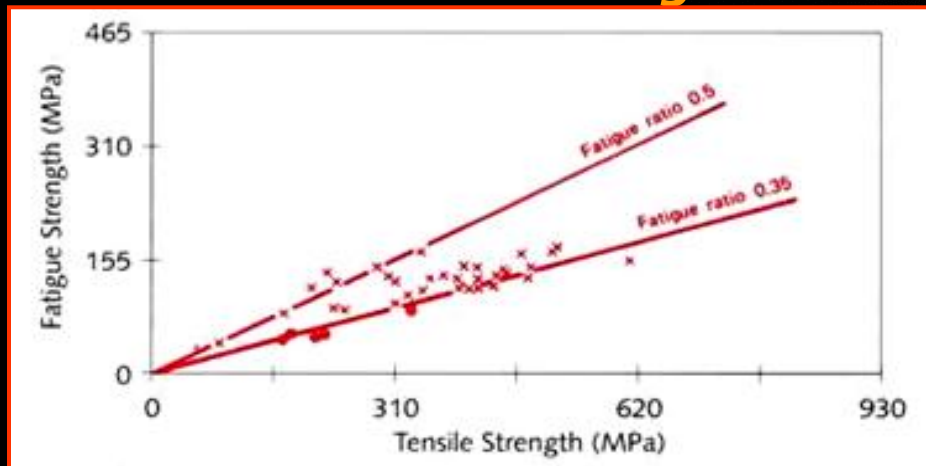
$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 200 \text{ ksi (1400 MPa)} \\ 100 \text{ ksi} & S_{ut} > 200 \text{ ksi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

Cast iron

$$S'_e = \begin{cases} 0.4S_{ut} & S_{ut} < 60 \text{ ksi (400 MPa)} \\ 24 \text{ ksi} & S_{ut} \geq 60 \text{ ksi} \\ 160 \text{ MPa} & S_{ut} < 400 \text{ MPa} \end{cases}$$



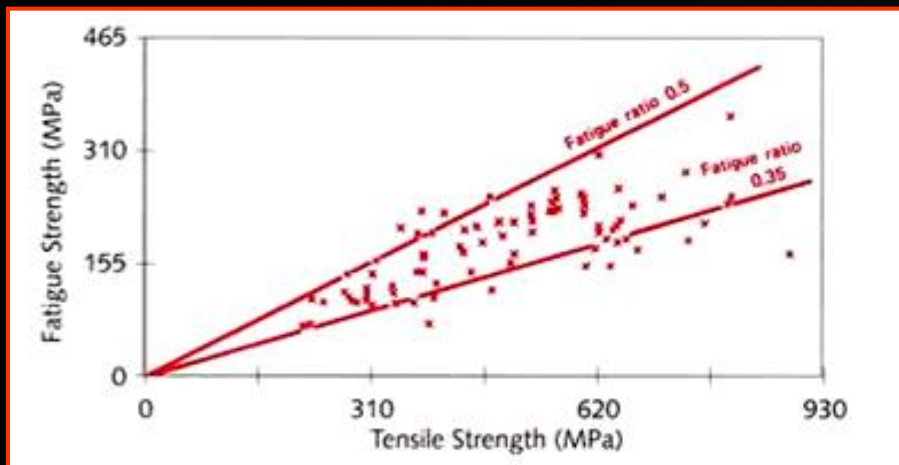
Relationship Between Endurance Limit and Ultimate Strength



Aluminum

$$S'_e = \begin{cases} 0.4S_{ut} & S_{ut} < 48 \text{ ksi (330 MPa)} \\ 19 \text{ ksi} & S_{ut} \geq 48 \text{ ksi} \\ 130 \text{ MPa} & S_{ut} \geq 330 \text{ MPa} \end{cases}$$

For $N = 5 \times 10^8$ cycle



Copper alloys

$$S'_e = \begin{cases} 0.4S_{ut} & S_{ut} < 40 \text{ ksi (280 MPa)} \\ 14 \text{ ksi} & S_{ut} \geq 40 \text{ ksi} \\ 100 \text{ MPa} & S_{ut} \geq 280 \text{ MPa} \end{cases}$$

For $N = 5 \times 10^8$ cycle

Correction Factors for Specimen's Endurance Limit

$$S_e = k_a k_b k_c k_d k_e k_f S_e'$$

Where S_e = endurance limit of component

S_e' = endurance limit experimental

k_a = surface finish factor (machined parts have different finish)

k_b = size factor (larger parts greater probability of finding defects)

k_c = reliability / statistical scatter factor (accounts for random variation)

k_d = loading factor (differences in loading types)

k_e = operating T factor (accounts for diff. in working T & room T)

k_f = stress concentration factor

Correction Factors for Specimen's Endurance Limit

- surface factor, k_a

The rotating beam test specimen has a polished surface. Most components do not have a polished surface. Scratches and imperfections on the surface act like stress raisers and reduce the fatigue life of a part. Use either the graph or the equation with the table shown below.

$$k_a = A (S_{ut})^b$$

Table 6-3 Coefficients for the Surface-Factor Equation

Source: Shigley and Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill, New York, 1989, p. 283 with permission

Surface Finish	MPa		kpsi	
	A	b	A	b
Ground	1.58	-0.085	1.34	-0.085
Machined or cold-rolled	4.51	-0.265	2.7	-0.265
Hot-rolled	57.7	-0.718	14.4	-0.718
As-forged	272	-0.995	39.9	-0.995

Correction Factors for Specimen's Endurance Limit

- Size factor, k_b

Larger parts fail at lower stresses than smaller parts. This is mainly due to the higher probability of flaws being present in larger components.

For round cross section

$$d \leq 0.3 \text{ in. (8 mm)} \quad k_b = 1$$

$$0.3 \text{ in.} < d \leq 10 \text{ in.} \quad k_b = .869(d)^{-0.097}$$

$$8 \text{ mm} < d \leq 250 \text{ mm} \quad k_b = 1.189(d)^{-0.097}$$

If the component is larger than 10 in., use $k_b = .6$

Correction Factors for Specimen's Endurance Limit

- Reliability factor, k_c

The reliability correction factor accounts for the scatter and uncertainty of material properties (endurance limit).

Reliability %	C_{reliab}
50	1.000
90	0.897
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

Correction Factors for Specimen's Endurance Limit

- Load factor, k_d

Pure bending $k_d = 1$

Pure axial $k_d = 0.7$

Pure torsion $k_d = 1$ if von Mises stress is used, use
0.577 if von Mises stress is NOT used.

Combined loading $k_d = 1$

❖ Operating temperature factor

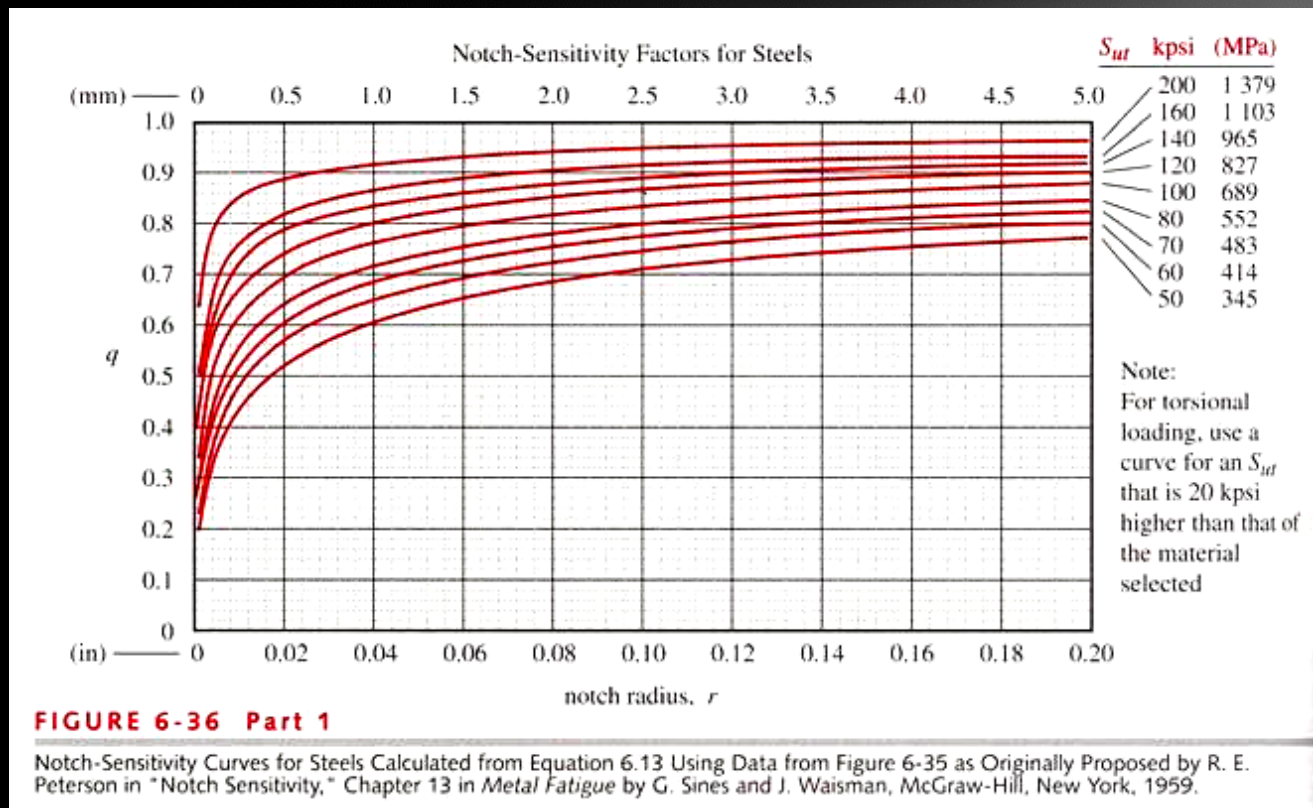
- Accounts for the difference between the test temperature and operating temperature of the component
- For carbon and alloy steels, fatigue strength not affected by operating temperature – 45 to 450°C
 $k_e = 1$
- At higher operating temperature
- $k_e = 1 - 5800(T - 450)$ for T between 450 and 550°C, or
- $k_e = 1 - 3200(T - 840)$ for T between 840 and 1020°F

Fatigue Stress Concentration Factor, K_f

Experimental data shows that the actual stress concentration factor is not as high as indicated by the theoretical value, K_t . The stress concentration factor seems to be sensitive to the notch radius and the ultimate strength of the material.

$$K_f = 1 + (K_t - 1)q$$

Notch sensitivity factor



Fatigue Stress Concentration Factor, K_f for Aluminum

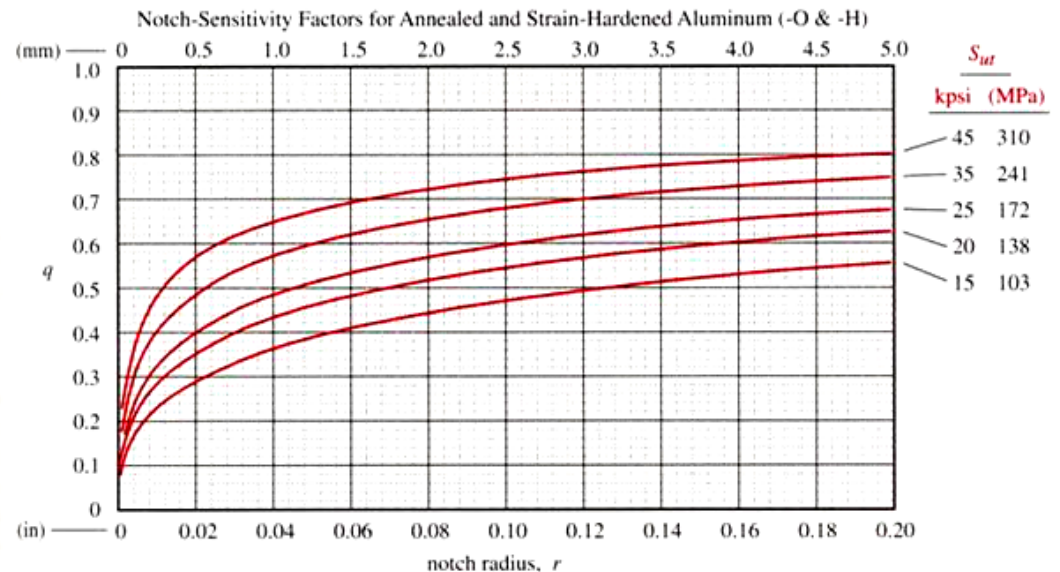
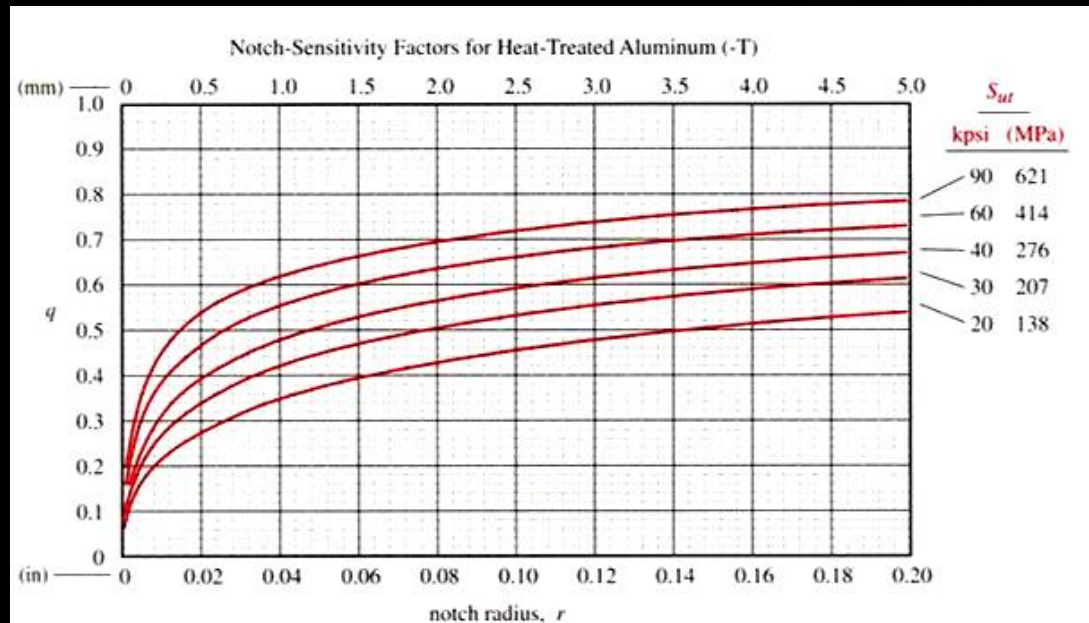


FIGURE 6-36 Part 2

Notch-Sensitivity Curves for Aluminums Calculated from Equation 6.13 Using Data from Figure 6-35 as Originally Proposed by R. E. Peterson in "Notch Sensitivity," Chapter 13 in *Metal Fatigue* by G. Sines and J. Waisman, McGraw-Hill, New York, 1959.

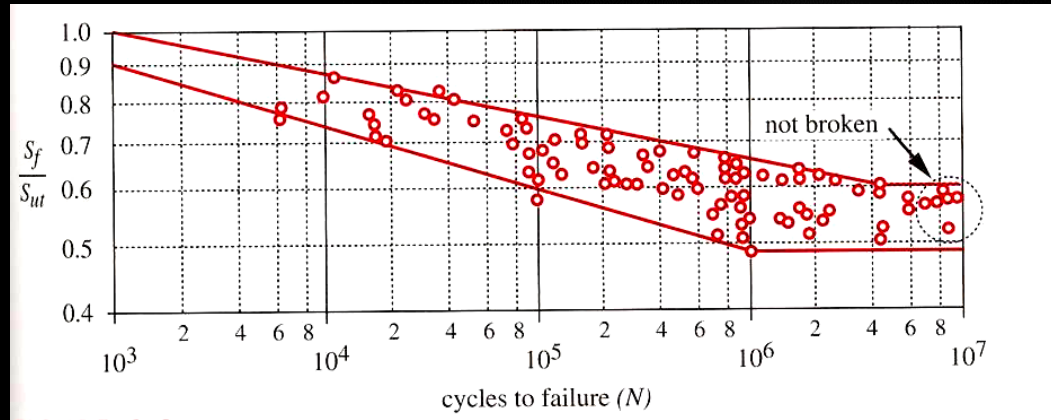
Design process – Fully Reversed Loading for Infinite Life

- Determine the maximum alternating applied stress, σ_a , in terms of the size and cross sectional profile
- Select material $\rightarrow S_y, S_{ut}$
- Choose a safety factor $\rightarrow n$
- Determine all modifying factors and calculate the endurance limit of the component $\rightarrow S_e$
- Determine the fatigue stress concentration factor, K_f
- Use the design equation to calculate the size

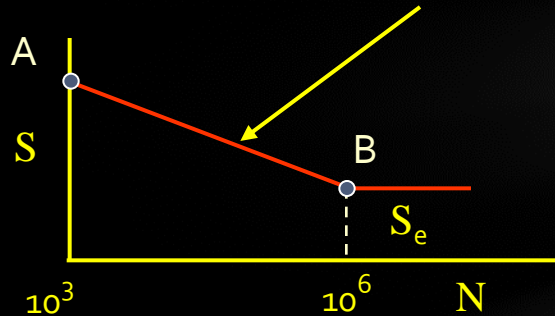
$$K_f \sigma_a = \frac{S_e}{n}$$

- Investigate different cross sections (profiles), optimize for size or weight
- You may also assume a profile and size, calculate the alternating stress and determine the safety factor. Iterate until you obtain the desired safety factor

Design for Finite Life

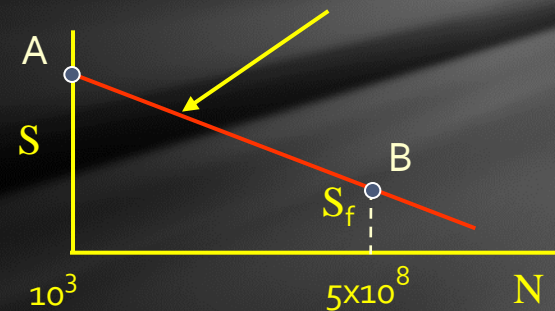


$S_n = a (N)^b$ equation of the fatigue line



Point A $\left\{ \begin{array}{l} S_n = .9S_{ut} \\ N = 10^3 \end{array} \right.$

Point B $\left\{ \begin{array}{l} S_n = S_e \\ N = 10^6 \end{array} \right.$



Point A $\left\{ \begin{array}{l} S_n = .9S_{ut} \\ N = 10^3 \end{array} \right.$

Point B $\left\{ \begin{array}{l} S_n = S_f \\ N = 5 \times 10^8 \end{array} \right.$

Design for Finite Life

$$S_n = a (N)^b$$

$$\log S_n = \log a + b \log N$$

Apply conditions for point A and B to find the two constants "a" and "b"

$$\left\{ \begin{array}{l} \log .9S_{ut} = \log a + b \log 10^3 \\ \log S_e = \log a + b \log 10^6 \end{array} \right. \longrightarrow \begin{array}{l} a = \frac{(.9S_{ut})^2}{S_e} \\ b = -\frac{1}{3} \log \left(\frac{.9S_{ut}}{S_e} \right) \end{array}$$

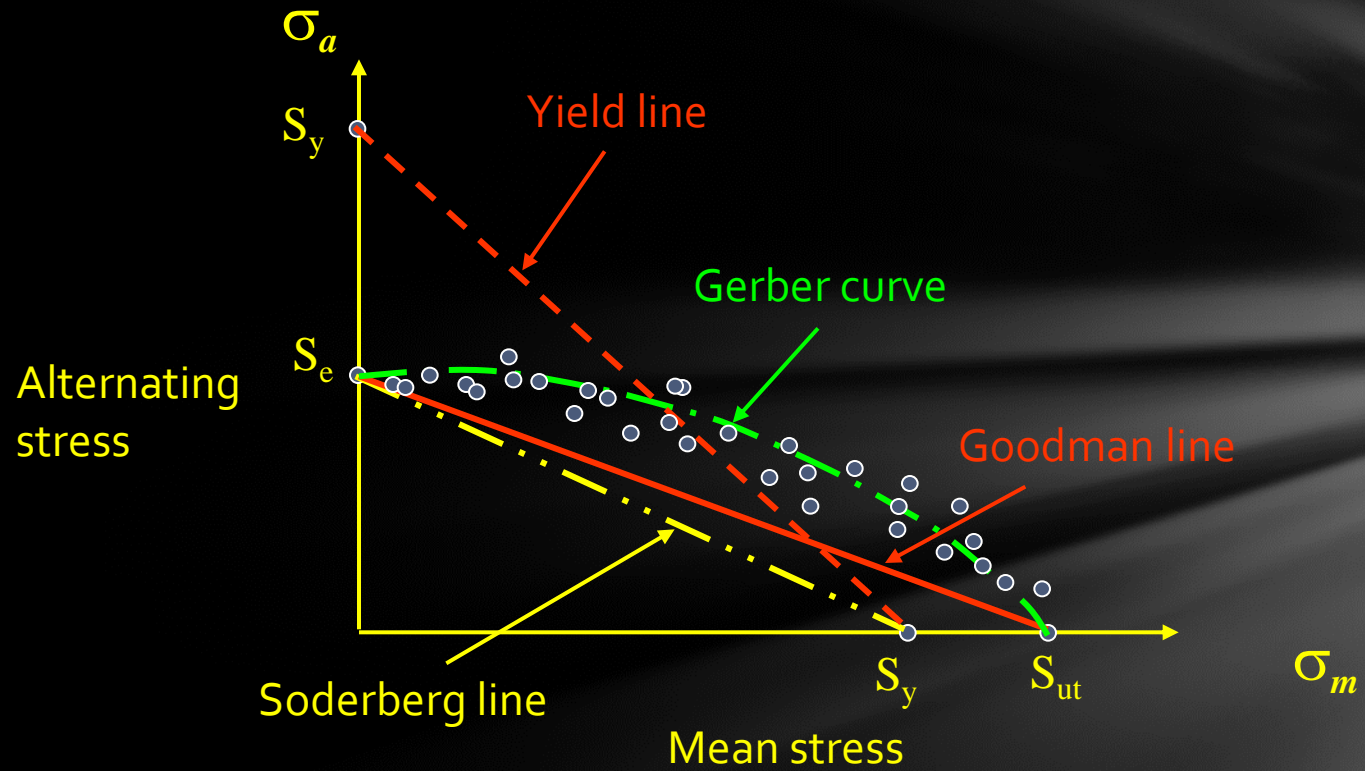
$$S_n = S_e \left(\frac{N}{10^6} \right)^{1/3 \log \left(\frac{S_e}{.9S_{ut}} \right)}$$

Calculate S_n and replace S_e in the design equation

$$K_f \sigma_a = \frac{S_n}{n} \quad \text{Design equation}$$

Fluctuating stresses

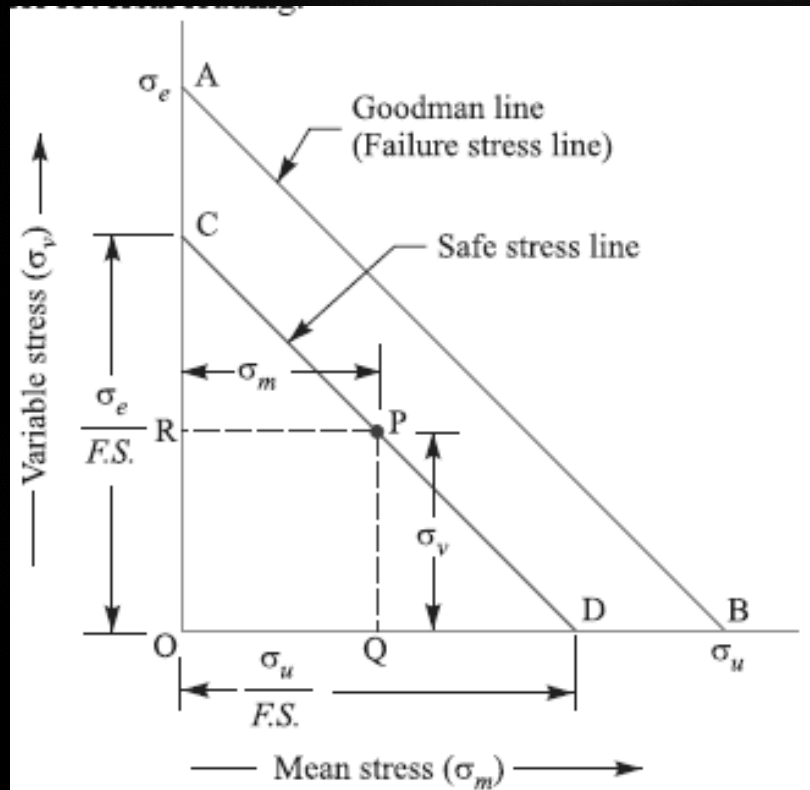
- The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in figure as functions of stress amplitude(σ_a) and mean stress (σ_m).
- The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile.
- In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.



Goodman Method for Combination of Stresses:

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line AB in figure given below follows the suggestion of Goodman.

A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.



Now from similar triangles COD and PQD,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \quad \dots(\because QD = OD - OQ)$$

$$\therefore \frac{* \sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

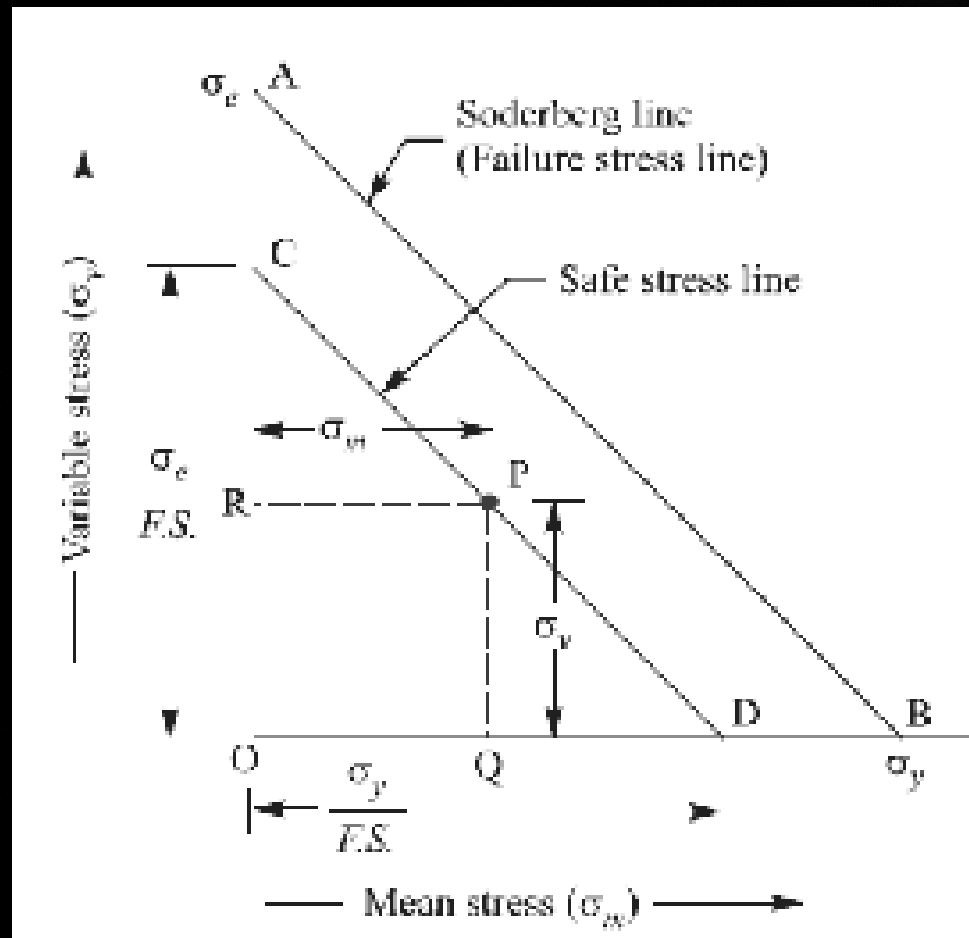
$$\sigma_v = \frac{\sigma_e}{F.S.} \left[1 - \frac{\sigma_m}{\sigma_u / F.S.} \right] = \sigma_e \left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u} \right]$$

$$\text{or} \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \quad \dots(i)$$

Soderberg Method for Combination of Stresses

- A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line AB in following figure, follows the suggestion of Soderberg line.
- This line is used when the design is based on yield strength. the line AB connecting σ_e and σ_y , as shown in following figure, is called ***Soderberg's failure stress line***.

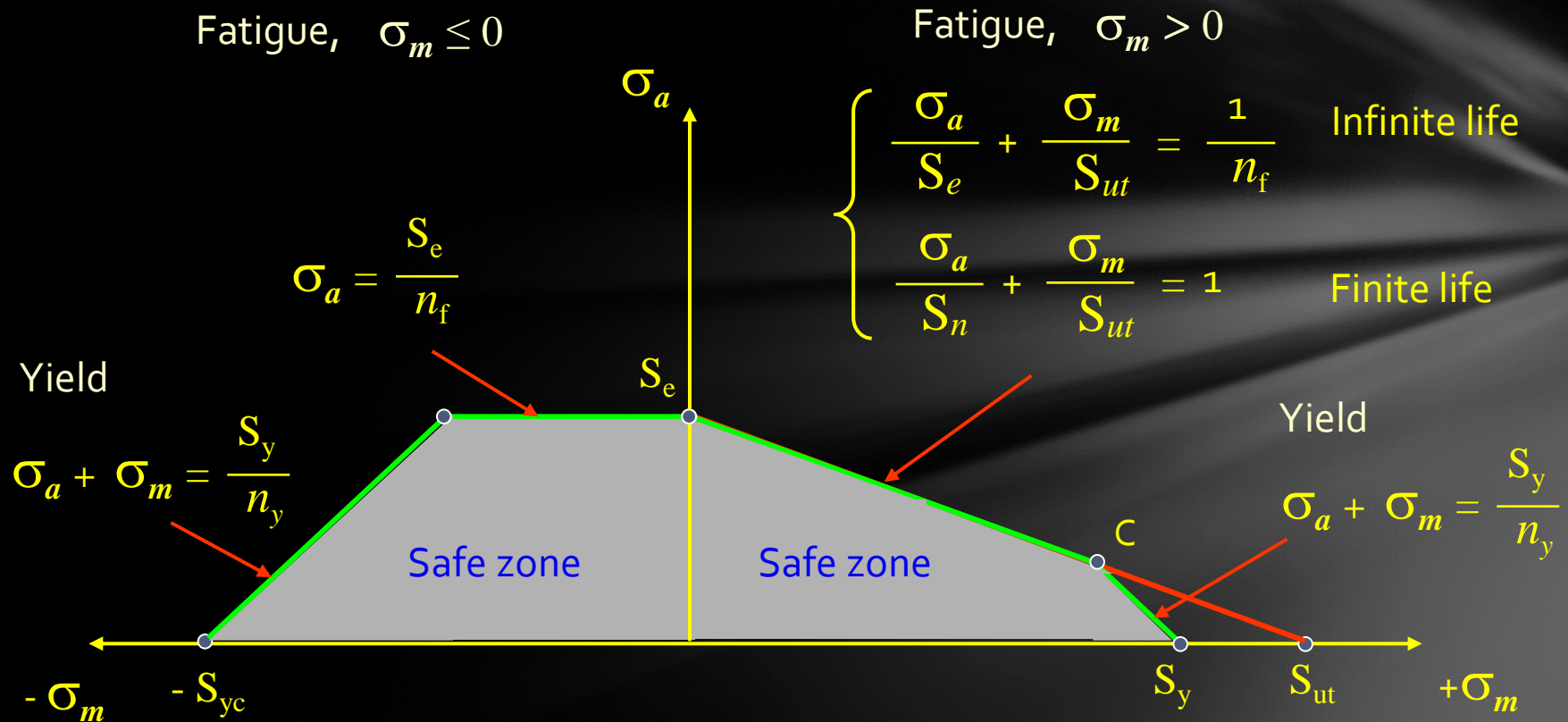
If a suitable factor of safety ($F.S.$) is applied to the endurance limit and yield strength, a safe stress line CD may be drawn parallel to the line AB .



Modified Goodman Diagram:

- In the design of components subjected to fluctuating stresses, the Goodman diagram is slightly modified to account for the yielding failure of the components, especially, at higher values of the mean stresses.
- The diagram known as modified Goodman diagram and is most widely used in the design of the components subjected to fluctuating stresses.

Modified Goodman Diagram for fluctuating axial and bending stresses



Combined Loading

All four components of stress exist,

σ_{xa} alternating component of normal stress

σ_{xm} mean component of normal stress

τ_{xya} alternating component of shear stress

τ_{xym} mean component of shear stress

Calculate the alternating and mean principal stresses,

$$\sigma_{1a}, \sigma_{2a} = (\sigma_{xa}/2) \pm \sqrt{(\sigma_{xa}/2)^2 + (\tau_{xya})^2}$$

$$\sigma_{1m}, \sigma_{2m} = (\sigma_{xm}/2) \pm \sqrt{(\sigma_{xm}/2)^2 + (\tau_{xym})^2}$$

Combined Loading

Calculate the alternating and mean von Mises stresses,

$$\sigma'_a = (\sigma_{1a}^2 + \sigma_{2a}^2 - \sigma_{1a}\sigma_{2a})^{1/2}$$

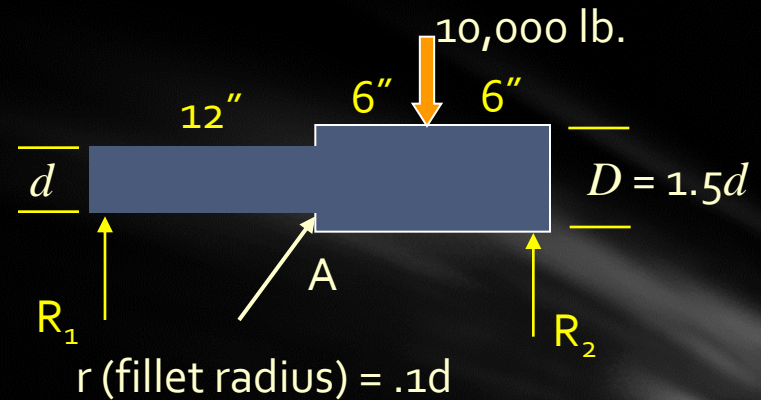
$$\sigma'_m = (\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m}\sigma_{2m})^{1/2}$$

Fatigue design equation

$$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n_f} \quad \text{Infinite life}$$

Design Example

A rotating shaft is carrying 10,000 lb force as shown. The shaft is made of steel with $S_{ut} = 120$ ksi and $S_y = 90$ ksi. The shaft is rotating at 1150 rpm and has a machine finish surface. Determine the diameter, d , for 75 minutes life. Use safety factor of 1.6 and 50% reliability.



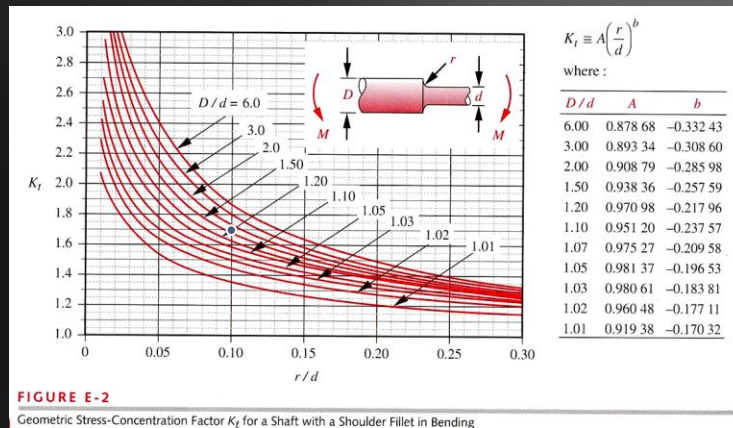
Calculate the support forces, $R_1 = 2500$, $R_2 = 7500$ lb.

The critical location is at the fillet, $M_A = 2500 \times 12 = 30,000$ lb-in

Calculate the alternating stress, $\sigma_a = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{305577}{d^3}$ $\sigma_m = 0$

Determine the stress concentration factor

$$\left\{ \begin{array}{l} \frac{r}{d} = .1 \\ \frac{D}{d} = 1.5 \end{array} \right. \longrightarrow K_t = 1.7$$



Design Example

Assume $d = 1.0$ in

Using $r = .1$ and $S_{ut} = 120$ ksi,
 q (notch sensitivity) = .85

$$K_f = 1 + (K_t - 1)q = 1 + .85(1.7 - 1) = 1.6$$

Calculate the endurance limit

$$C_{load} = 1 \text{ (pure bending)}$$

$$C_{rel} = 1 \text{ (50\% rel.)}$$

$$C_{temp} = 1 \text{ (room temp)}$$

$$C_{surf} = A (S_{ut})^b = 2.7(120)^{-0.265} = .759$$

$$0.3 \text{ in.} < d \leq 10 \text{ in.} \quad C_{size} = .869(d)^{-0.097} = .869(1)^{-0.097} = .869$$

$$S_e = C_{load} C_{size} C_{surf} C_{temp} C_{rel} (S'_e) = (.759)(.869)(.5 \times 120) = 39.57 \text{ ksi}$$

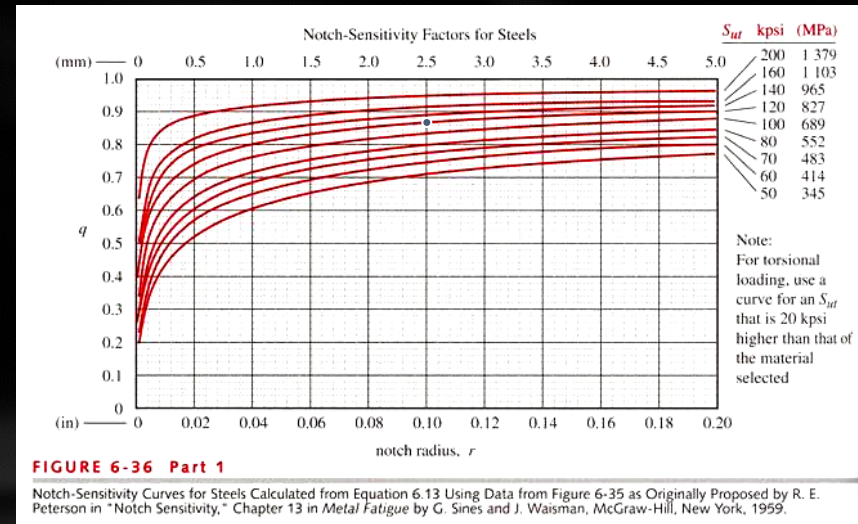


Table 6-3 Coefficients for the Surface-Factor Equation

Source: Shigley and Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill, New York, 1989, p. 283 with permission

Surface Finish	MPa		kpsi	
	A	b	A	b
Ground	1.58	-0.085	1.34	-0.085
Machined or cold-rolled	4.51	-0.265	2.7	-0.265
Hot-rolled	57.7	-0.718	14.4	-0.718
As-forged	272	-0.995	39.9	-0.995

Design Example

Design life, $N = 1150 \times 75 = 86250$ cycles

$$S_n = S_e \left(\frac{N}{10^6} \right)^{1/3} \log \left(\frac{S_e}{.9 S_{ut}} \right) \quad S_n = 39.57 \left(\frac{86250}{10^6} \right)^{1/3} \log \left(\frac{39.57}{.9 \times 120} \right) = 56.5 \text{ ksi}$$

$$\sigma_a = \frac{305577}{d^3} = 305.577 \text{ ksi} \quad n = \frac{S_n}{K_f \sigma_a} = \frac{56.5}{1.6 \times 305.577} = .116 < 1.6$$

So $d = 1.0$ in. is too small

Assume $d = 2.5$ in

All factors remain the same except the size factor and notch sensitivity.

Using $r = .25$ and $S_{ut} = 120$ ksi,
 q (notch sensitivity) = .9

$$K_f = 1 + (K_t - 1)q = 1 + .9(1.7 - 1) = 1.63$$

$$C_{size} = .869(d)^{-0.097} = .869(2.5)^{-0.097} = .795 \quad \rightarrow \quad S_e = 36.2 \text{ ksi}$$

Design Example

$$S_e = 36.2 \text{ ksi} \rightarrow S_n = 53.35 \text{ ksi}$$

$$\sigma_a = \frac{305577}{(2.5)^3} = 19.55 \text{ ksi}$$

$$n = \frac{S_n}{K_f \sigma_a} = \frac{53.35}{1.63 \times 19.55} = 1.67 \approx 1.6$$

$$d = 2.5 \text{ in.}$$

Check yielding

$$n = \frac{S_y}{K_f \sigma_{max}} = \frac{90}{1.63 \times 19.55} = 2.8 > 1.6 \quad \text{okay}$$

THANK YOU